

Lecture on Beam Theory

Leonardo da Vinci, 1452-1519

Perhaps the smartest person ever

Strikingly handsome

Great physical strength

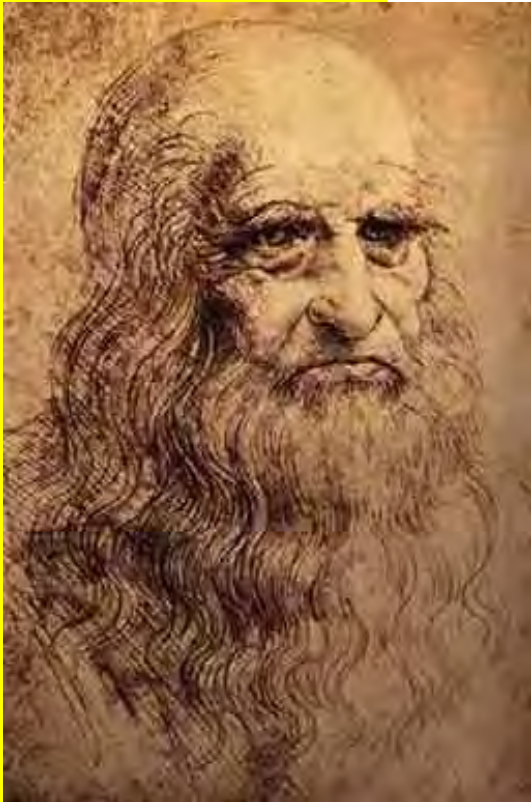
Fine singing voice

Vegetarian with strict dietary rules

*Loved animals; bought caged animals
at markets and set them free*

Unrepentant left-handed

Wrote in mirror image





La Gioconda (Mona Lisa)
Le Louvre, Paris



*The Virgin and Child with St. Anne
(Le Louvre; recently restored)*

Jacques Castaing, Louvre Museum





***Study for the head of Christ for the Last Supper
(Pinacoteca di Brera, Milano)***



***Head of the Virgin in Three-Quarter View Facing Right
(Harris Brisbane Dick Fund)***

The manuscripts:

Codex Arundel (British Museum, London)

Codex Atlanticus (Biblioteca Ambrosiana, Milan)

Codex Trivulzianus (Biblioteca Trivulziana, Milan)

Codex 'On the Flight of Birds' (Biblioteca Reale, Turin)

Codex Ashburnham (Institute de France, Paris)

Codices of the Institut de France

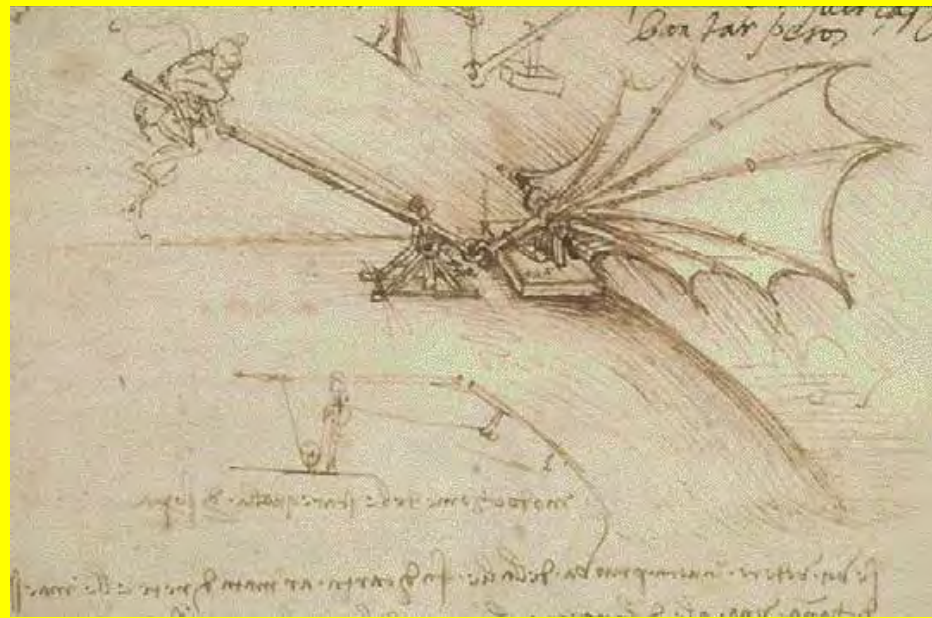
Codex Forster (Victoria and Albert Museum, London)

Codex Leicester (Bill Gates)

Windsor folios (Windsor Castle, Royal Collection)

The Madrid Codices (National Library of Madrid)

Note that Genevra de Benci is in the National Gallery of Art in D.C.

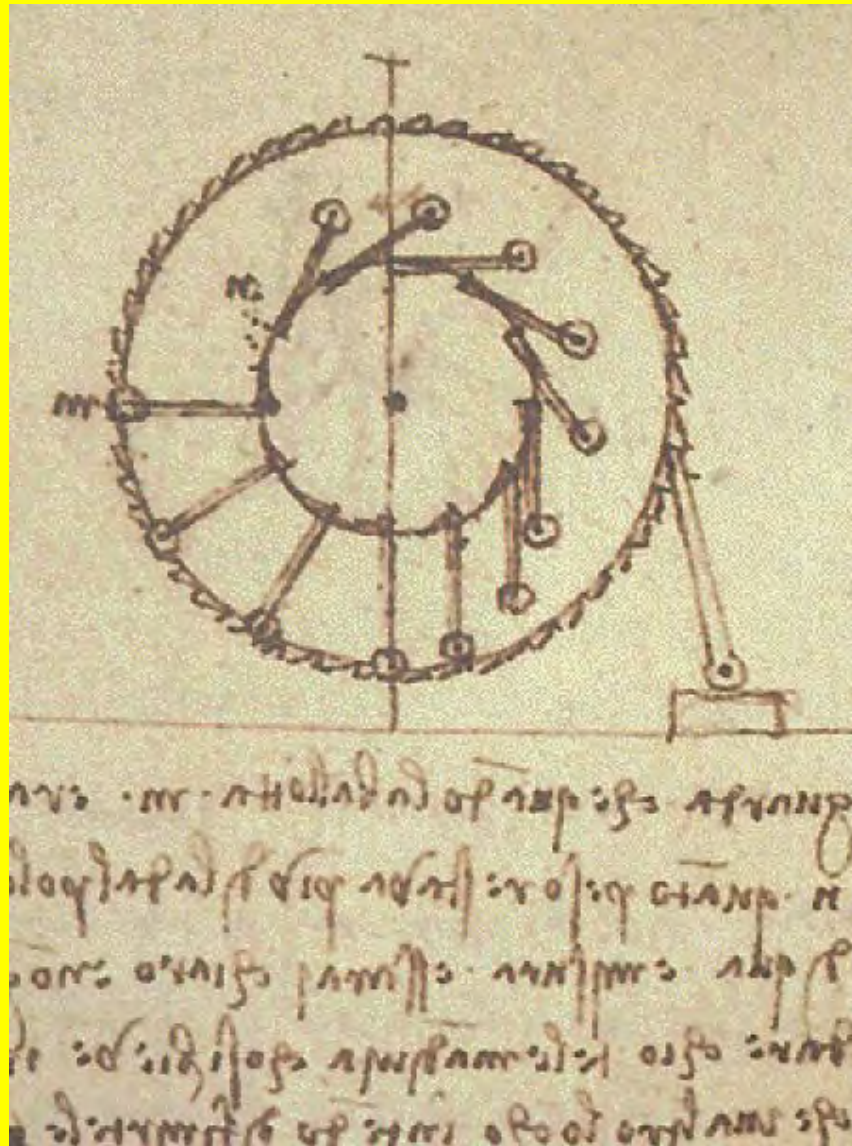


*Device for testing beating wings
(Manuscript B, folio 88 v.)*



Parachute

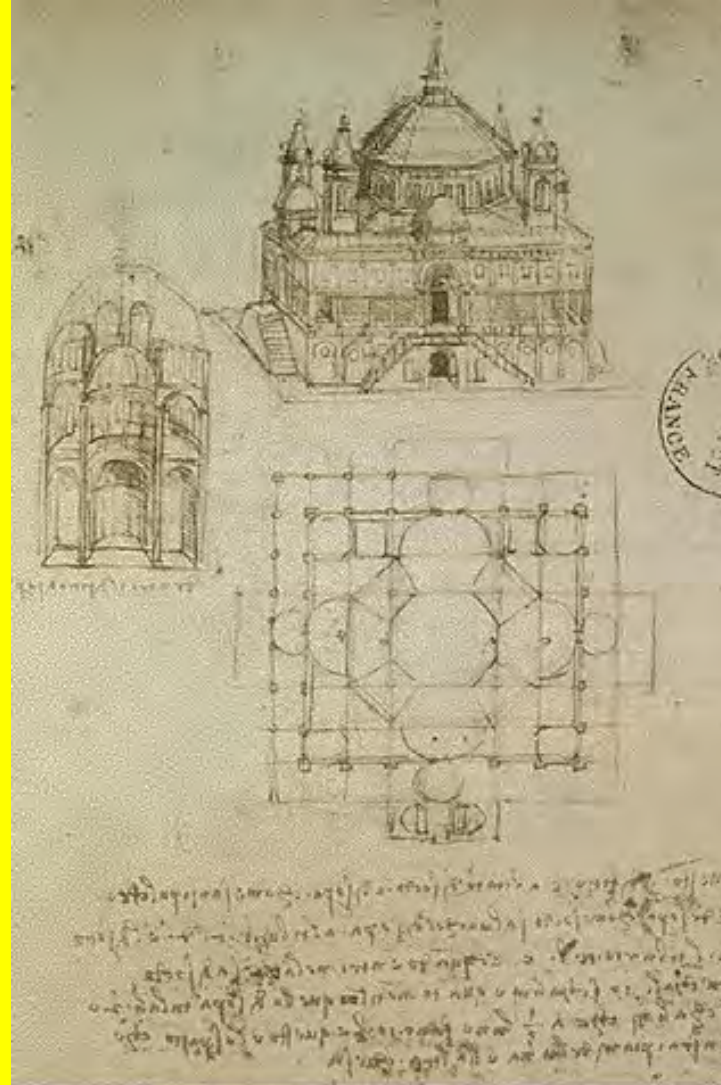
(Codex Atlanticus, folio 1058, Biblioteca Ambrosiana di Milano)



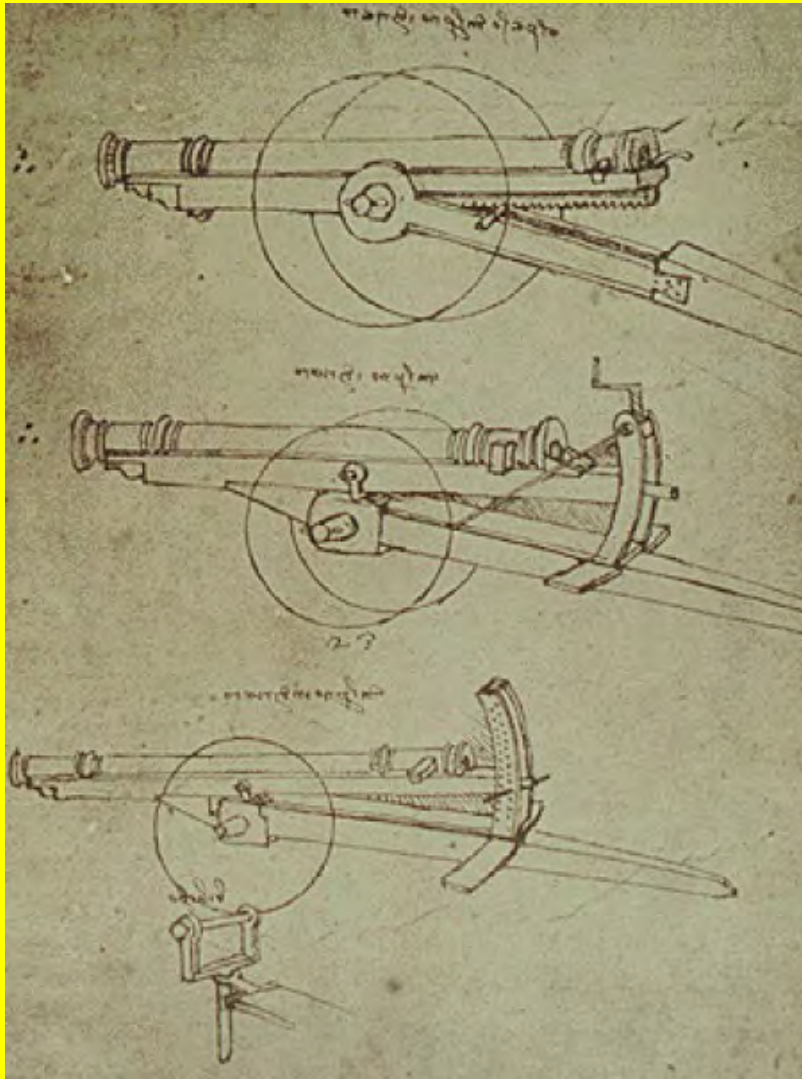
*Wheel for studies on the impossibility of perpetual motion
(Codex Forster II, folio 90 v., Victoria and Albert Museum, London)*



***Lagoon Dredge
(Manuscript E, folio 75 v.)***



*From Codex Ashburnham
(Institute de France, Paris)*



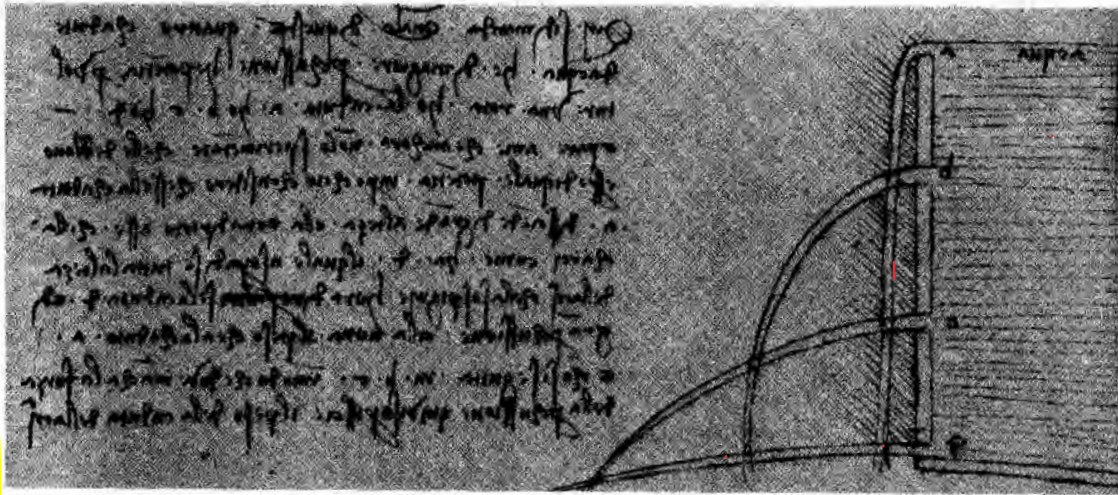
*Firearm with screw elevating gear
(Codex Atlanticus, folio 76 v.
Biblioteca Ambrosiana di Milano)*

Here the question is asked: which of these four waterfalls has more percussion and power in order to turn a wheel: fall a or b, c or d? I have not yet experimented, but it seems to me that they must have the same power, considering that a, even if it descends from a great height, has no other water chasing it, as has d, which bears upon itself the whole height of the thrusting water. Now, if fall d has a great percussion, it has not the weight of fall a. And the same is true for b and c. Consequently, where the force of percussion is lacking, it is compensated by the weight of the waterfall.



Leonardo introduces the terms “potentia” (energy), “weight” (potential energy) and “percussion” (kinetic energy). He concludes that the potentia is the same for each of the four waterfalls; the Bernoulli equation!

$$p + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$$



His conclusions in these studies also imply the validity of the law governing the speed of falling bodies, a law deduced by Galileo's experiments and enunciated by Torricelli in 1642.

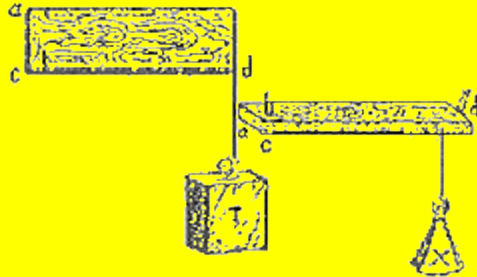
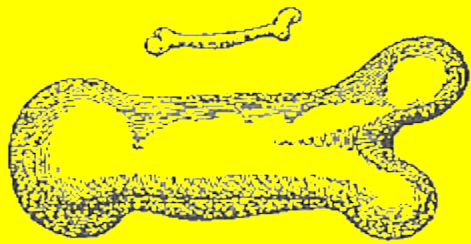
Carlo Zammattio, in Mechanics of Water and Stone, The Unknown Leonardo, McGraw Hill Co., New York, 1974 (Codex Madrid, folio 134 v.)

Château d'Amboise

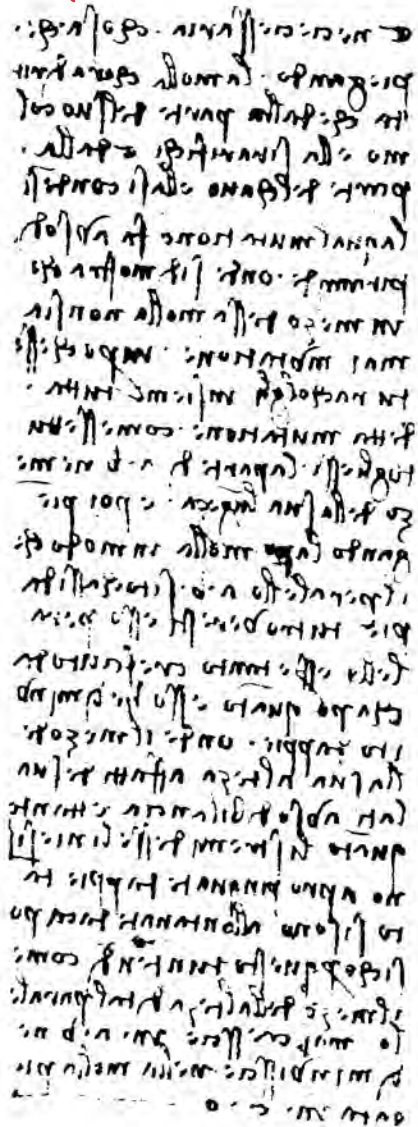


Chapelle St. Hubert





Galileo Galilei (Pisa, 1564 - Arcetri, 1642): *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica ed i movimenti locali* (Leiden, 1638).

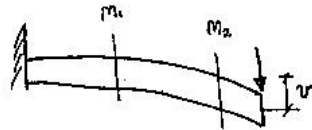
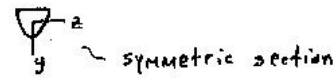
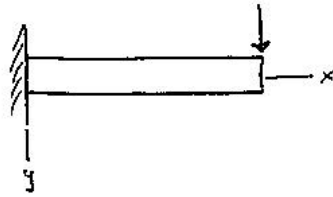


If a straight spring is bent, it is necessary that its convex part become thinner and its concave part, thicker. This modification is pyramidal, and consequently there will never be a change in the middle of the spring. You shall discover, if you consider all of the aforementioned modifications, that by taking part ab in the middle of its length and then bending the spring in a way that the two parallel lines, a and b, touch at the bottom, the distance between the parallel lines has grown as much at the top as it has diminished at the bottom. Therefore, the center of its height has become much like a balance for the sides. And the ends of those lines draw as close at the bottom as much as they draw away at the top. From this, you will understand why the center of the height of the parallels never increases in ab nor diminishes in the bent spring at co.

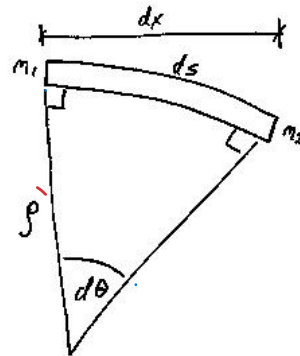
**Carlo Zammattio, in
Mechanics of Water
and Stone, *The
Unknown Leonardo*,
McGraw Hill Co.,
New York, 1974**

STRESSES IN BEAMS

Review: Normal stresses



x - y plane is plane of bending



$\rho \equiv$ radius of curvature

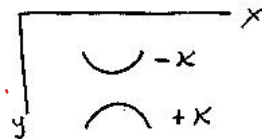
$\kappa \equiv \frac{1}{\rho} \equiv$ curvature

$$\rho d\theta = ds$$

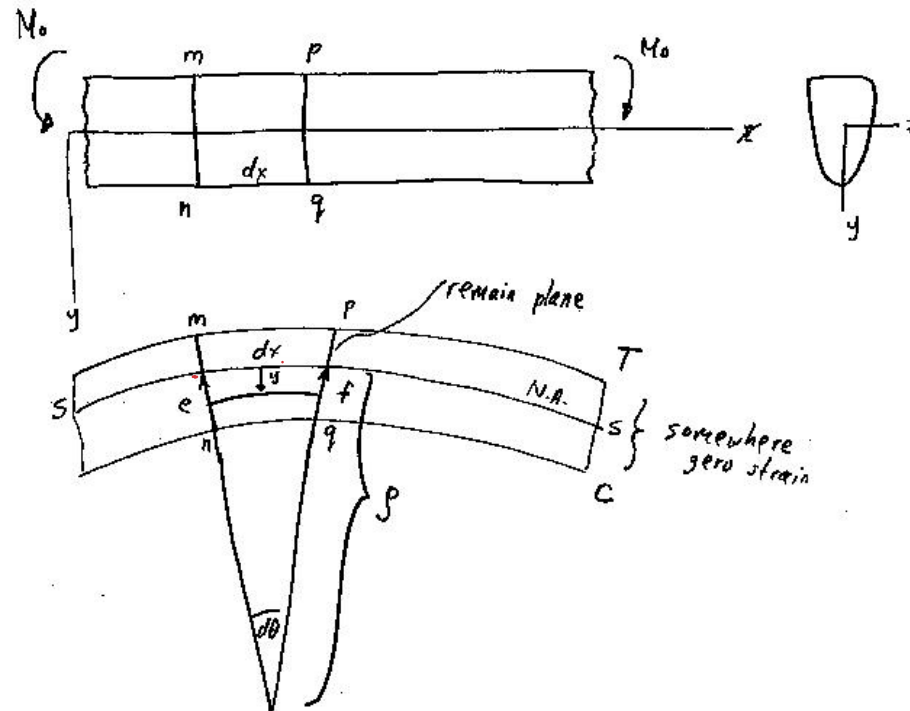
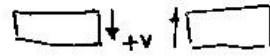
$ds \approx dx$ for small v

$$\rho d\theta = dx$$

$$\boxed{\frac{1}{\rho} = \kappa = \frac{d\theta}{dx}}$$



Convention



at N.A. length r.t.s. $\rho d\theta = dx$

length of ef is $(\rho - y)d\theta = dx - \frac{y}{\rho} dx$

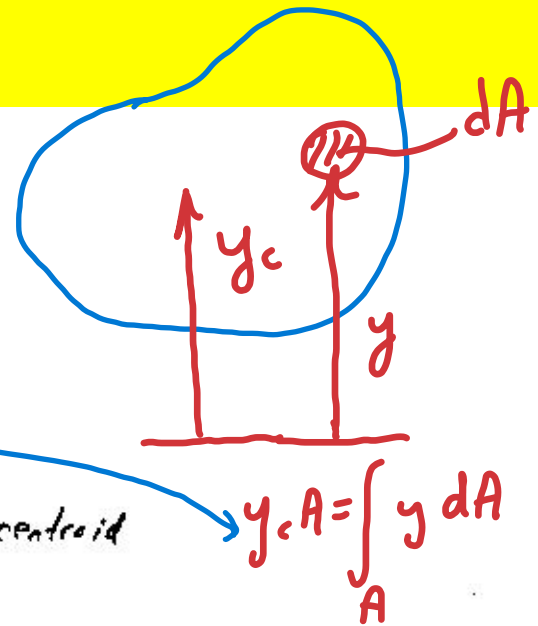
change in length is $-\frac{y}{\rho} dx$

$$\epsilon_x = -\frac{y}{\rho} = -\kappa y$$

$$\sigma_x = E \epsilon_x = -E x y$$

$$\sum F_x = 0 \quad \int_A \sigma_x dA = -E x \underbrace{\int_A y dA}_0 = 0$$

\therefore N.A. passes thru centroid



$$y_c A = \int_A y dA$$

$$M = \int_A \sigma_x y dA = -E x \int_A y^2 dA = -x E I_{zz}$$

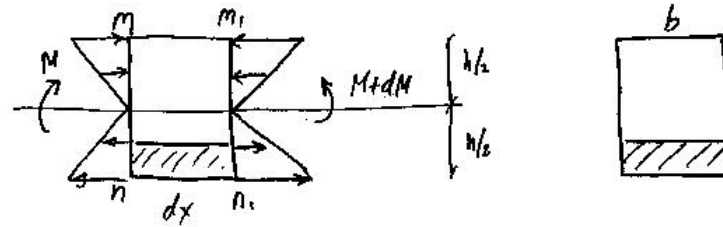
$$x = \frac{1}{\rho} = -\frac{M}{EI}$$

$$\boxed{\sigma_x = \frac{M y}{I}}$$

$$\boxed{EI \frac{y''}{(1+(y')^2)^{3/2}} = -M}$$

$$u = \frac{1}{2} \sigma \epsilon = \frac{1}{2} \frac{\sigma^2}{E} = \frac{1}{2E} \frac{M^2 y^2}{I^2}$$

Shear stresses:



$$dF_1 = \sigma_x dA = \frac{My}{I} dA$$

$$F_1 = \int \frac{My}{I} dA$$

$$F_2 = \int \frac{(M+dM)y}{I} dA$$

$$F_3 = \tau b dx$$

$$\sum F_x = 0 \quad F_3 = F_2 - F_1$$

$$\tau b dx = \int \frac{(M+dM)y}{I} dA - \int \frac{My}{I} dA$$

$$\tau = \frac{dM}{dx} \frac{1}{Ib} \int y dA$$

$$\tau = \frac{VQ}{Ib}$$