Some Highlights of Arch Analysis

In 1675, Robert Hooke (1635-1703) discovered an announced the analogy between an arch and a hanging chain.

As he did for "Hooke's Law", he published the result in anagram form:

"ut pendet continuum flexile, sic stabit contiguum rigidum inversum"

("as hangs the flexible line, so but inverted will stand the rigid arch")

Meaning:

Both the arch and the chain are in equilibrium. The chain can support only tension, and the masonry arch acts in compression. (The forces are reversed).

Generally, the shape a chain assumes under a set of forces, if made rigid and inverted, illustrates a path of compressive forces for an arched structure to support the same loads. This shape is called the funicular shape for the load system.

Hooke's discovery was used by Giovanni Poleni (1683-1761), an Italian mathematician who worked on hydraulics, physics, astronomy, etc. to assess the safety of St. Peter's dome, which had developed numerous cracks.

Poleni divided the dome into slices and hung 32 unequal weights proportional to the weight of the corresponding sections of each "arch" wedge

He then showed that the hanging chain could fit within the section of the arch, indicating the dome was safe.

Important result:

if a line of thrust can be achieved within the arch, then the arch is safe. How safe it is requires advanced analysis that is beyond the scope of the course.

SWITCH TO

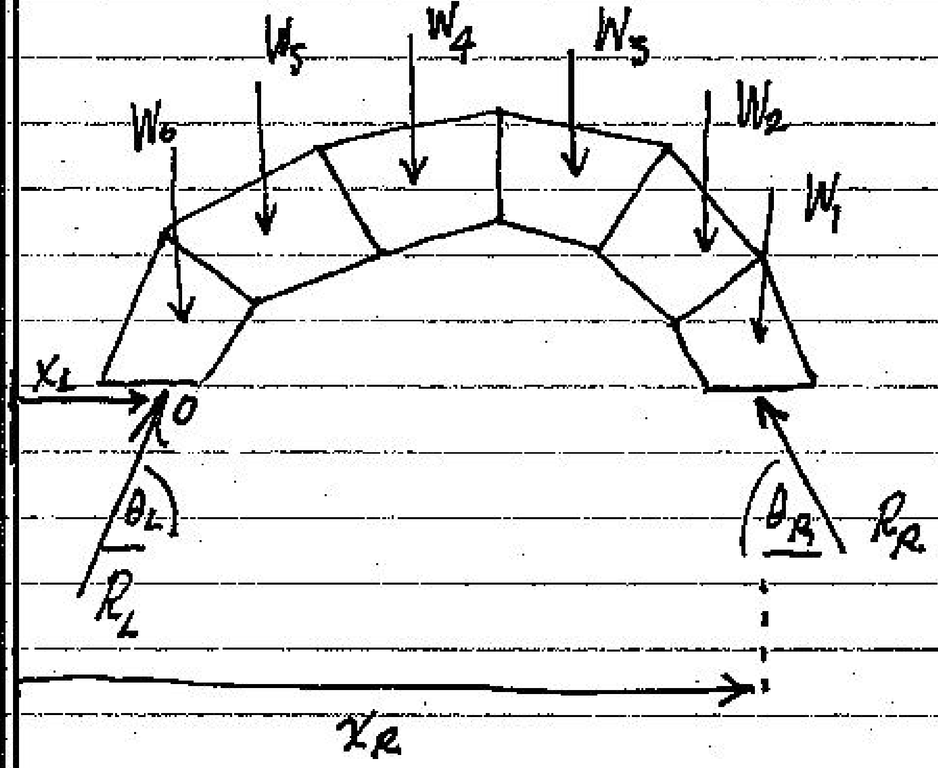
web.mit.edu/masonry/interactiveThrust

for examples

Random Segmented Arch

Assumptions

1. No tension across the joints
2. Voussoirs are rigid (won't break nor deform)
3. Sufficient friction μ
4. Thrust shown can be achieved



Let us determine how many unknowns there are:

$x_L,\ x_R,\ \theta_L,\ \theta_R,\ R_L,\ R_R$

We have 3 eqn's:

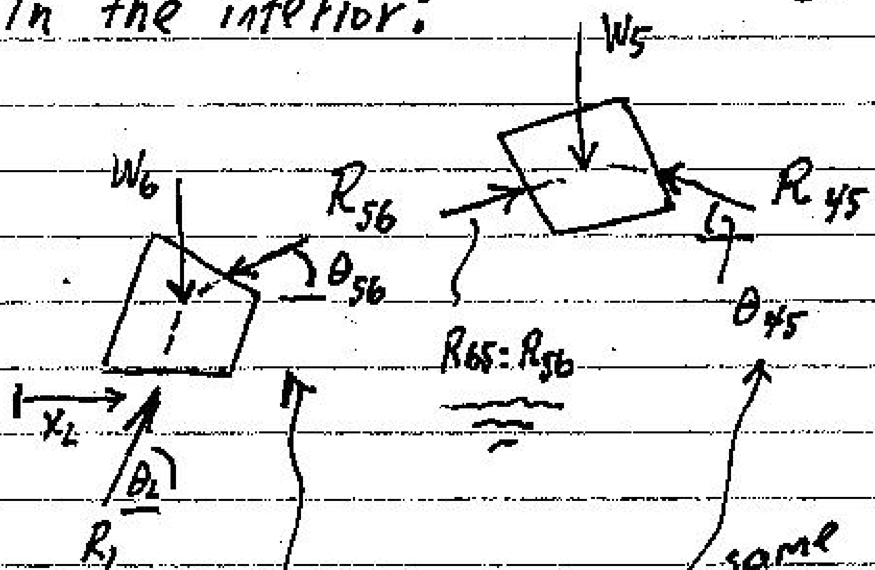
$$\Sigma F_x = 0 \qquad R_L \cos\theta_L = R_R \cos\theta_R$$

$$\Sigma F_y = 0 \qquad R_L \sin\theta_L + R_R \sin\theta_R = \sum_{i=1}^{6} W_i$$

$$\Sigma M_0 = 0 \qquad R_R \sin\theta_R \,(x_R - x_L) = \sum_{i=1}^{6} W_i (\bar{x}_i - x_L)$$

Keep $\underbrace{x_L, R_L, \theta_L \text{ as unknowns}}_{\text{indeterminate to third degree}}$

In the interior:



additional 2 unk., but have 2 additional eqn's

same here

Thus we see that if we knew what are x_L, R_L, θ_L, then we can march along and solve for all forces and angles (and lines of action!!)

Let us assume a trial value of the set

x_L, R_L, θ_L

If we find a sol'n to the problem, we will note that the directions of the internal forces will define a "line of thrust"