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The interaction between a crack and a dislocation dipole

ROBERTO BALLARINI[1] and MITSUNORI DENDA[2]
[1]Department of Civil Engineering, Case Western Reserve University, Cleveland, OH 44106, USA;
[2]Department of Mechanics and Materials Science, Rutgers University, Piscataway, NJ 08855-0909, USA

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Abstract. This paper presents the results of an analysis which considers the interaction between a semi-infinite crack and a dislocation dipole. Applying the operator derived by Denda [1] to the crack/dislocation interaction solution developed by Lo [2], explicit expressions are obtained for the stress intensity factors at the tip of the crack. Results are computed and discussed for a variety of geometrical configurations, with the intent of developing an understanding of the effects of position and orientation of the dislocation dipole on crack tip shielding and antishielding. The solution can be used as a Green's function in semi-empirical analyses such as the one proposed by Chudnovsky [3], where the interaction between a crack in a polymeric material and the damage (crazing) which surrounds it is solved by experimentally measuring the crack opening displacements of the crazes and calculating the amount of toughening caused by the damage.

1. Introduction

It is well known that in many materials crack growth is accompanied by the formation of damage (microcracking, crazing, etc.) around the main crack. Because this damage can constitute an important toughening mechanism, the elastic interaction of a crack with an array of microcracks has been addressed by various authors. The mathematical treatment of the problem of interaction between random cracks in a three dimensional body is very complicated. Therefore, most of the solutions that have been developed reduce the problem to two dimensions. Approximate solutions to the problem which neglect the interaction between the cracks were developed by Panasjuk [4] (only pairs of cracks interact) as well as Carpinteri et al. [5] (none of the cracks interact), in their analyses of a thin plate with random cracks subjected to biaxial tension. Their analyses assume that propagation of the most dangerous crack leads to failure of the solid. The failure load was determined using fracture mechanics formulas. The interaction between many cracks can in principle be reduced to a system of singular integral equations which represent appropriate traction boundary conditions along the surfaces of the cracks. However, since these equations require a numerical solution, the interaction between more than a few cracks cannot be calculated. To circumvent this problem several approximate techniques have been proposed in an attempt to gain insight into the interaction between several microcracks and a main crack. Rose [6] developed a very interesting solution where the microcracks are represented as point sources whose strengths are determined using a self-consistent scheme. The solution to the problem involves solving a system of algebraic equations, thus eliminating the need to solve a system of integral equations. However, the algebra involved in setting up the equations is quite laborious, and, as pointed out by Rose, the method becomes inpractical when the number of microcracks becomes large. Chudnovsky [7, 8] has also proposed a selfconsistent procedure which is valid for three-dimensional problems as well as two-dimensional problems based on the combination of the double layer potential (dislocation dipole) technique and Willis' polynomial conservation theorem. Again, the computations involved in solving the problem become prohibitive when the number of cracks becomes large. Rubinstein [9] performed an exact stress analysis of the interaction between a two-dimensional microcrack and a semi-infinite main crack for the special case where the cracks are collinear. His results are significant since they provided a basis for checking the previously described approximate solutions. While the solutions described above have led to significant qualitative understanding of the effects of several microcracks on a main crack, it can be concluded that the problem of elastic interaction between many cracks is still unsolved.

In their study of fracture propagation in polystyrene, Chudnovsky and Ben Ouezdou [10] approached the problem of the interaction between damage and a main crack from a different viewpoint. Realizing that for a random configuration of a large number of microcracks the solutions which satisfy traction-free boundary conditions on all cracks involve extremely tedious and time consuming numerical procedures, they proposed to solve the interaction problem in a semi-empirical manner. In their analysis an approximate integral representation for the stress intensity factors produced by the damage was developed in terms of the crack opening displacements of the crazes surrounding the main crack (dislocation dipole approach), and by employing experimentally observed crack opening displacements, they calculated the amount of toughening induced by the damage around the main crack. The solution was approximate because the Green's function they used did not satisfy all of the boundary conditions. However, their approach offers considerable promise for quantifying the toughening produced by the damage, since it eliminates the need for satisfying boundary conditions along the large number of crazes (it might be added that the boundary conditions along the crazes are not known anyway, since the surfaces of the cracks are not necessarily stress free).

In this paper the Green's function which represents the stress intensity factor produced by a dislocation dipole on a main crack is presented. This function satisfies the traction-free boundary conditions on the surfaces of the main crack, and can be used in the semi-empirical analysis by integration along paths which represent the damage (crazing). This solution is practical because the integration can be performed using simple quadrature and does not involve solving a large number of algebraic equations.

2. Method of analysis

The analysis is based on a straightforward extension of the results obtained by Lo [2] considering the interaction of a dislocation and a crack. As shown by Lo, the stress and displacement fields in the vicinity of a crack interacting with an edge dislocation located at an arbitrary point $z_0 = x_0 + \mathrm{i}y_0$ (Fig. 1) are given by

$$
\begin{aligned}
\sigma_{xx} + \sigma_{yy} &= 4\,\mathrm{Real}\,(\Phi(z)) \\
\sigma_{yy} - \sigma_{xx} + 2\mathrm{i}\sigma_{xy} &= 2[\bar{z}\Phi'(z) + \Psi(z)] \qquad (1) \\
2\mu(u' + \mathrm{i}v') &= \kappa\Phi(z) - z\overline{\Phi'(z)} - \overline{\Phi(z)} - \overline{\Psi(z)},
\end{aligned}
$$

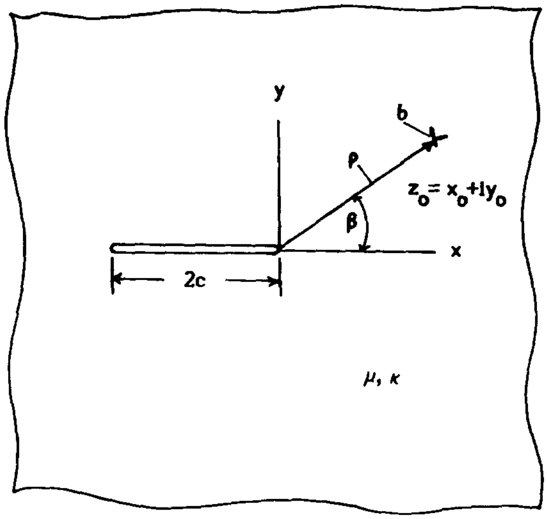

Fig. 1. Dislocation interacting with a crack.

where

$$
\begin{aligned}
\Phi &= \Phi_1 + \Phi_2 \\
\Psi &= \Psi_1 + \Psi_2 \\
\Phi_1(z; z_0) &= \alpha/(z - z_0) \\
\Psi_1(z; z_0) &= \bar{\alpha}/(z - z_0) + \alpha\bar{z}_0/(z - z_0)^2 \\
\Phi_2(z; z_0) &= -[\alpha F(z; z_0) + \alpha F(z; \bar{z}_0) + \bar{\alpha}(z_0 - \bar{z}_0)G(z; \bar{z}_0) - \alpha X(z)] \\
\Psi_2(z; z_0) &= \overline{\Phi_2(\bar{z})} - \Phi_2(z) - z\Phi_2'(z)
\end{aligned}
\tag{2}
$$

and

$$
\begin{aligned}
F(z; z_0) &= \frac{1}{2}\left[1 - \frac{X(z)}{X(z_0)}\right]\Big/(z - z_0) \\
G(z; z_0) &= \frac{\partial}{\partial z_0} F(z; z_0) \\
X(z) &= z^{-1/2}(z + 2c)^{-1/2}.
\end{aligned}
\tag{3}
$$

The dislocation data are included in the constant α, defined by the relation

$$
\alpha = \frac{\mu b}{\pi \mathrm{i}(\kappa + 1)} \tag{4}
$$

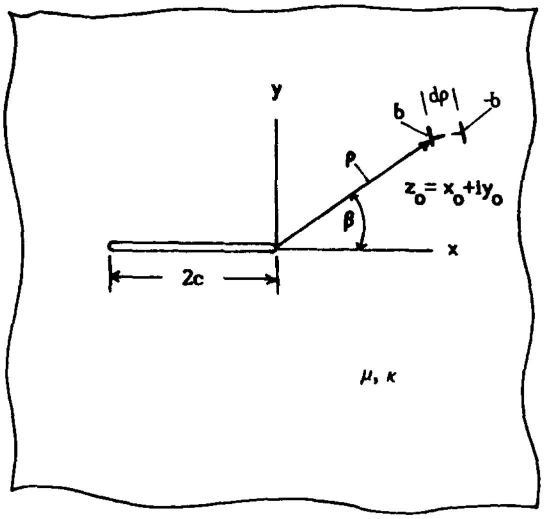

Fig. 2. Dislocation dipole interacting with a crack.

in which μ is the shear modulus, κ is related to Poisson's ratio ($\kappa = 3 - 4\nu$ for plane strain; $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress), and $b = e^{i\theta}\{[u_r] + i[v_\theta]\} = b_x + ib_y$ is the Burgers vector ($[x]$ denotes the jump in the quantity x).

The potentials Φ and Ψ, as defined in (2)-(4), satisfy the traction-free boundary condition on the crack surface. They were derived using the formalism and techniques developed by Muskhelishvili [11], based on the properties of Cauchy integrals and piecewise analytic functions.

It should be noted that in the present work the interest lies in assessing the influence of a dislocation dipole, therefore terms representing the external loading have been omitted. These can be included using the principle of superposition.

As shown by Denda [1], the interaction between a dislocation dipole and a crack (Fig. 2) can be obtained by the superposition of the solutions to a pair of edge dislocations, with Burgers vectors of the same magnitude but of the opposite directions separated by an infinitesimally small distance $d\varrho$. This leads to [1]

$$\Phi_d(z; z_0, \bar{z}_0) = \frac{\partial \Phi}{\partial z_0} e^{i\theta} d\varrho + \frac{\partial \Phi}{\partial \bar{z}_0} e^{-i\theta} d\varrho, \tag{5}$$

where Φ_d and Ψ_d are the complex potentials which represent the solution to the dislocation dipole problem, and Φ and Ψ are given by (2).

Application of the differential operator (5) to the limit as $|z_0|/2c$ approaches zero of the Green's functions for a discrete dislocation interacting with a crack leads to the following expressions for the Green's functions for a dislocation dipole interacting with a semi-infinite crack.

$$\Phi_d = \Phi_{d_1} + \Phi_{d_2}$$

$$\Psi_d = \Psi_{d_1} + \Psi_{d_2}$$

$$\Phi_{d_1}(z; z_0) = \alpha \frac{\partial}{\partial z_0} [(z - z_0)^{-1}] \, d\varrho \, e^{i\theta}$$

$$\Psi_{d_1}(z; z_0, \bar{z}_0) = \alpha \bar{z}_0 \frac{\partial}{\partial z_0} [(z - z_0)^{-2}] \, d\varrho \, e^{i\theta}$$

$$+ (\bar{\alpha} \, d\varrho \, e^{i\theta} + \alpha \, d\varrho \, e^{-i\theta}) \frac{\partial}{\partial z_0} [(z - z_0)^{-1}]$$

$$\Phi_{d_2}(z; z_0, \bar{z}_0) = -\frac{1}{2\sqrt{z}} \left\{ \alpha \frac{\partial}{\partial z_0} [(z^{1/2} + z_0^{1/2})^{-1}] \, d\varrho \, e^{i\theta} \right.$$

$$+ \alpha \frac{\partial}{\partial \bar{z}_0} [(z^{1/2} + \bar{z}_0^{1/2})^{-1}] \, d\varrho \, e^{-i\theta} \qquad (6)$$

$$+ \bar{\alpha}(e^{i\theta} - e^{-i\theta}) \frac{\partial}{\partial \bar{z}_0} [(z^{1/2} + \bar{z}_0^{1/2})^{-1}] \, d\varrho$$

$$\left. + \bar{\alpha}(z_0 - \bar{z}_0) \frac{\partial^2}{\partial \bar{z}_0^2} [(z^{1/2} + \bar{z}_0^{1/2})^{-1}] \, d\varrho \, e^{-i\theta} \right\}$$

$$\Psi_{d_2}(z) = \overline{\Phi_{d_2}(\bar{z})} - \Phi_{d_2}(z) - z\Phi'_{d_2}(z).$$

It should be noted that the limit, which represents the case for which the distance between the crack tip at $z = 0$ and the dislocation dipole is very small is taken to simplify the algebraic manipulations. These terms can be retained if one is interested in introducing the crack length as a parameter.

3. Stress intensity factor analysis

The stress and displacement fields can be determined from the potentials Φ_d and Ψ_d through (1). In particular, the stress intensity factors, defined by

$$K_{\mathrm{I}} - iK_{\mathrm{II}} = \lim_{\sqrt{z} \Rightarrow 0} \sqrt{2\pi z} \, (\sigma_{yy} - i\sigma_{xy}) \qquad (7)$$

become

$$K_{\mathrm{I}} - iK_{\mathrm{II}} = \varrho^{-3/2} \, d\varrho \left\{ \alpha \cos(\theta - 3\beta/2) - \frac{3i\bar{\alpha}}{2} \sin\beta \cos(\theta - 5\beta/2) \right.$$

$$\left. + i\bar{\alpha} \sin\theta \, e^{3i\beta/2} + \frac{3\bar{\alpha}}{2} \sin\beta \sin(5\beta/2 - \theta) \right\} (2\pi)^{1/2} \qquad (8)$$

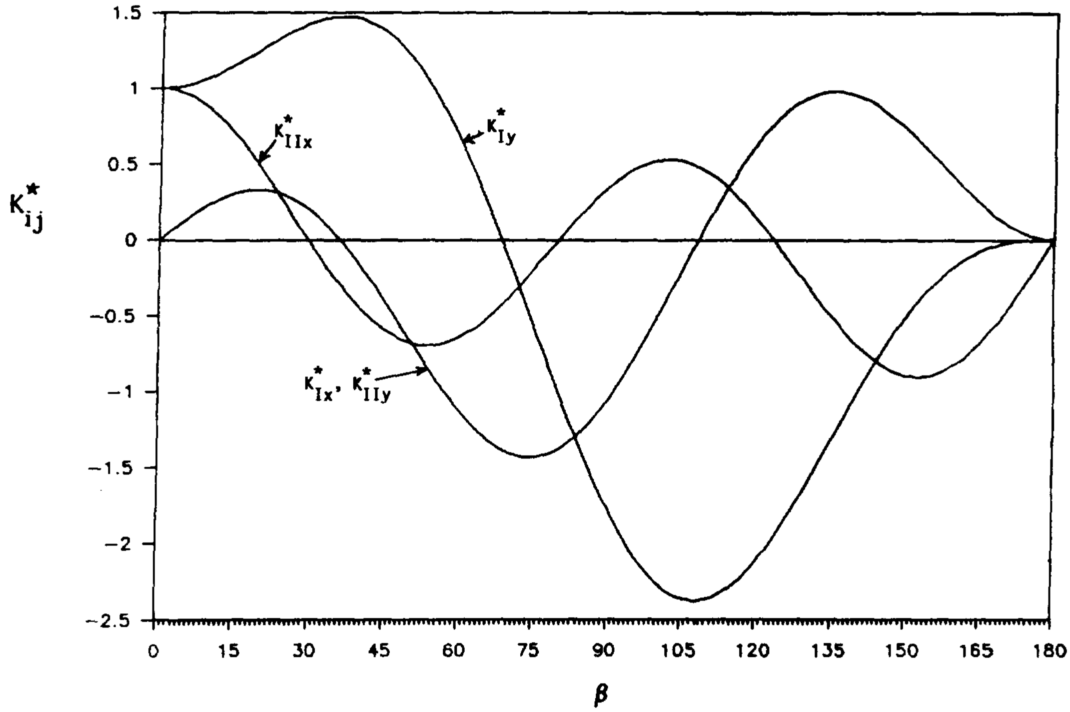

Fig. 3. Angular variation of stress intensity factors.

where $z_0 = \varrho\, e^{i\beta}$. For the special case where the dislocation dipole is parallel to the main crack ($\theta = 0$), (8) reduces to

$$K_\mathrm{I} - \mathrm{i}K_\mathrm{II} = \frac{\mu\, \mathrm{d}\varrho}{2\pi(\kappa + 1)\varrho^{3/2}} \{b_y[2 \cos (3\beta/2) + 3 \sin (5\beta/2) \sin \beta]$$

$$+ b_x[3 \sin \beta \cos (5\beta/2)] + \mathrm{i}\{b_y[-3 \sin \beta \cos (5\beta/2)]$$

$$+ b_x[3 \sin \beta \sin (5\beta/2) - 2 \cos (3\beta/2)]\}\}(2\pi)^{1/2} \tag{9}$$

Equation (9) is nondimensionalized by defining the following.

$$K_\mathrm{I}^* - \mathrm{i}K_\mathrm{II}^* = (\pi/2)^{1/2} \frac{(\kappa + 1)\varrho^{3/2}}{\mu\, \mathrm{d}\varrho\, |b|} (K_\mathrm{I} - \mathrm{i}K_\mathrm{II}), \tag{10}$$

where $|b| = (b_x^2 + b_y^2)^{1/2}$ is the magnitude of the Burgers vector.

The angular variation of these dimensionless stress intensity factors are shown in Fig. 3 for several combinations of b_x and b_y. In this plot $K_{ij}^*(i = \mathrm{I}, \mathrm{II}\ j = x, y)$ represents the stress intensity factor K_i^* due to b_j. The most interesting and valuable information obtained from these results is the shielding effect the dislocation dipole produces on the crack, especially the mode I intensity due to $b_y(b_x = 0)$. For $\beta < 68.4°$ this dipole contributes a positive K_I^* (amplification), while for $\beta > 68.4°$ it produces a negative K_I^* (shielding). The

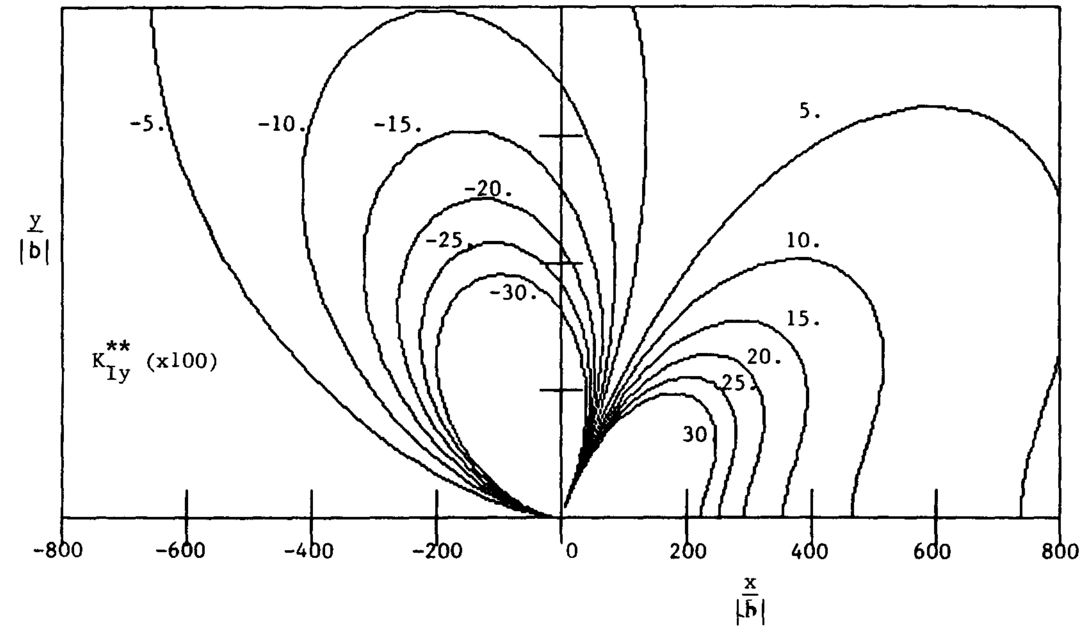

Fig. 4. Contours of equal levels of nondimensionalized mode I stress intensity factor due to b_y.

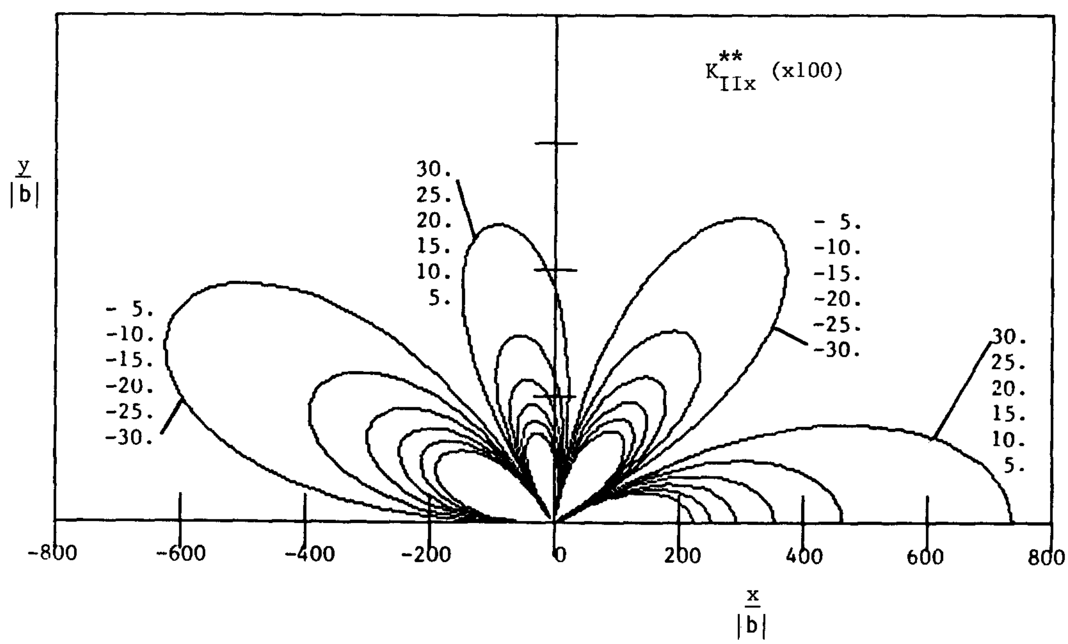

Fig. 5. Contours of equal levels of nondimensionalized mode II stress intensity factor due to b_x.

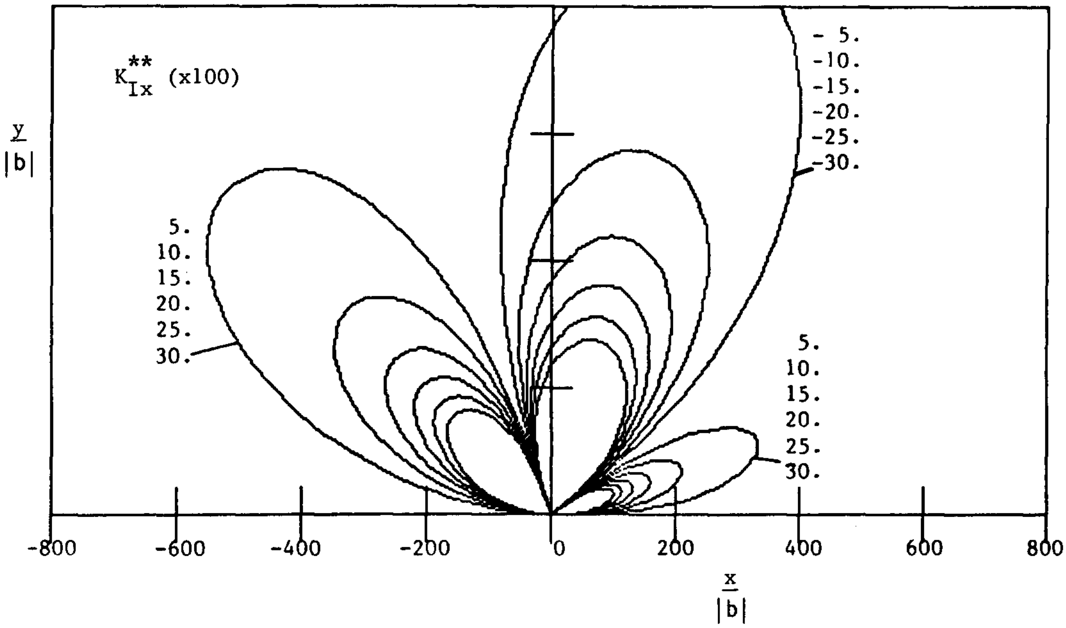

Fig. 6. Contours of equal levels of nondimensionalized mode I stress intensity factor due to b_x.

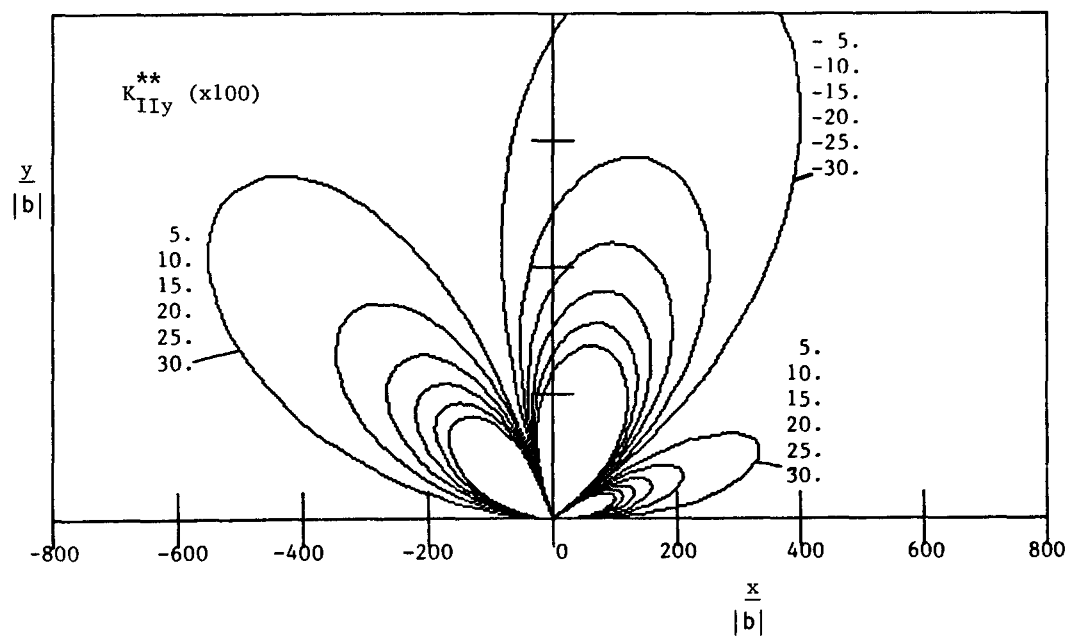

Fig. 7. Contours of equal levels of nondimensionalized mode II stress intensity factor due to b_y.

maximum value of the mode I stress intensity factor due to b_y occurs at 36 deg for anti-shielding and at 108 deg for shielding.

Figures 4–7 are contour plots of equal levels of the quantities

$$K_{\mathrm{I}}^{**} - \mathrm{i}K_{\mathrm{II}}^{**} = (\pi/2)^{1/2} \frac{(\kappa + 1)}{\mu \, \mathrm{d}\varrho} |b|^{1/2}(K_{\mathrm{I}} - \mathrm{i}K_{\mathrm{II}}) \tag{11}$$

as functions of the dimensionless distance $(\varrho/|b|)$, where K_{ij}^{**} represents $K_i^{**}(i = \mathrm{I, II})$ due to $b_j(j = x, y)$.

These results agree with those calculated by Chudnovsky and Ben Ouezdou, and are similar to those obtained by Rose in his study of a microcrack interacting with a main crack. It seems reasonable to expect similarities between the effects of a microcrack and a dislocation dipole, since it is well known that a crack can be modeled as a continuous distribution of dislocations (or dislocation dipoles).

4. Use of the Green's function in semi-empirical analyses

This section presents a brief description of how the Green's function for the stress intensity factor caused by a dislocation dipole can be utilized to quantify the effect of damage in a semi-empirical analysis.

Imagine an experiment being conducted on a cracked specimen, where the damage which evolves during crack propagation, which for this material consists of lines of discontinuity (crazes, for example), is monitored optically (the crack opening displacements of each line of discontinuity are measured). The stress intensity factor produced by the damage can be calculated by integrating (8) as follows:

$$\begin{aligned}(K_{\mathrm{I}} - \mathrm{i}K_{\mathrm{II}}) = \int_\Gamma \{&\alpha(\varrho, \beta) \cos [\theta(\varrho, \beta) - 3\beta/2] \\ &- \frac{3\mathrm{i}}{2} \overline{\alpha(\varrho, \beta)} \sin \beta \cos [\theta(\varrho, \beta) - 5\beta/2] \\ &+ \mathrm{i}\overline{\alpha(\varrho, \beta)} \sin \theta(\varrho, \beta) \, \mathrm{e}^{3\mathrm{i}\beta/2} \\ &+ \frac{3}{2} \overline{\alpha(\varrho, \beta)} \sin \beta \sin [5\beta/2 - \theta(\varrho, \beta)]\} (2\pi)^{1/2} \varrho^{-3/2} \, \mathrm{d}\varrho,\end{aligned} \tag{12}$$

where Γ are all paths representing the lines of discontinuity (damage). As a simple example consider a craze with a constant opening b_y located ahead of the crack tip as shown in Fig. 8. For this case (12) can be integrated in closed form. This results in

$$K_{\mathrm{I}} = \frac{2\mu(2/\pi)^{1/2}}{(\kappa + 1)} b_y \{\varrho_1^{-1/2} - \varrho_2^{-1/2}\}, \tag{13}$$

where ϱ_1 and ϱ_2 are the left and the right coordinates of the craze, respectively.

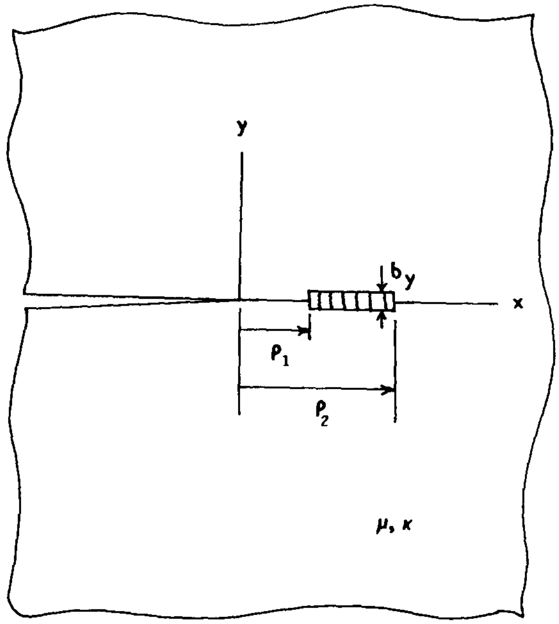

Fig. 8. Craze interacting with a crack.

5. Conclusion

A Green's function has been derived for the stress intensity factors produced by the interaction between a dislocation dipole and a crack. This solution can be utilized in semi-empirical analyses to quantify the effects on a crack of the damage which evolves during its propagation. While the discussion has been limited to the case where the damage consists of infinitesimally thin discontinuities, the method could be generalized [1] to include damage which is continuous over an area.

Work is currently under way to gather experimental data using polystyrene as a model material. The experimentally measured crack opening displacements of the crazes surrounding a crack will be substituted into (12), and the equation will be solved numerically. This will quantify the effect of damage on the fracture behavior of the material. The results of this work will be presented in a future communication.

Acknowledgements

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Résumé. On présente les résultats d'une analyse prenant en compte l'interaction entre une fissure semi-infinie et une paire de dislocations. En appliquant un opérateur dû à Denda à la solution d'interaction fissure/dislocation proposée par Lo, on obtient des solutions explicites pour les facteurs d'intensité de contraintes à l'extrémité de la fissure. Les résultats sont calculés et discutés pour une gamme de configurations géométriques, dans le but de développer une intelligence des effets de la position et de l'orientation de la paire de dislocations sur l'effet de couverture ou de non-couverture à l'extrémité de la fissure. La solution peut être utilisée sous la forme d'une fonction de Green dans les analyses semi-empiriques telles que celles proposées par Chudnovsky, où l'interaction entre une fissure dans un matériau polymère et l'endommagement (criques) qui l'environne est résolue par une mesure expérimentale des COD des criques et par le calcul de l'intensité du durcissement associé à l'endommagement.