International Journal of Fracture **71**: 95–97, 1995.

K-dominance for a pressurized Griffith crack

ALBERTO ROMEO and ROBERTO BALLARINI
Department of Civil Engineering, Case Western Reserve University, Cleveland, Ohio 44106-7201, USA

Received 15 August 1994; accepted 24 January 1995

Abstract. This paper shows that the pressurized Griffith crack is associated with a relatively small region of K-dominance.

Introduction

Linear elastic fracture mechanics (LEFM) is based on small scale yielding arguments (SSY), which rely on the fact that the singular first term in the expansion of the stress field dominates in a region that extends a distance r_o from the crack tip. SSY states that if the irreversible deformations that lead to crack initiation are confined within a 'process zone' of size r_p that is sufficiently smaller than r_o, then initiation will occur when the stress intensity factor K, which represents the coefficient of the singular term, reaches a critical value K_c. Determining r_o for a given configuration and loading requires the solution of a boundary value problem, and analytic results are available only for relatively simple geometries and loadings. These include the classic Griffith crack loaded by constant stress at infinity, for which r_o is relatively large. In this brief note it is shown that the pressurized Griffith crack, which has the same K as the remotely stressed Griffith crack, is associated with a very small r_o. This last result suggests that a fracture criterion based on a critical stress intensity factor should be applied with caution to a pressure loaded crack, and may have significant implications in fields such as hydraulic fracture.

Analytic results for a Griffith crack

Consider the plane elastostatics problems shown in Fig. 1a. A traction free crack of length $2a = 2$ in an infinitely extended plate is subjected to *remote tension* $\sigma^{\infty} = 1$. The stress ahead of the crack tip is given by

$$\sigma_{yy}^{\mathrm{rem}} = \frac{1+r}{\sqrt{r(r+2)}}, \tag{1}$$

where r denotes the distance from the crack tip. If the crack surfaces are loaded by a *constant pressure* $p = 1$ (Fig. 1b), the stress ahead of the crack tip is given by

$$\sigma_{yy}^{\mathrm{press}} = \frac{1+r}{\sqrt{r(r+2)}} - 1. \tag{2}$$

For both loading conditions the stress intensity factor is $K = \sqrt{\pi}$.

For $r \ll 1$ Eqns. (1) and (2) are approximated by

$$\sigma_{yy}^{\mathrm{asymptotic\text{-}rem}} = \frac{1}{\sqrt{2r}} + \frac{3\sqrt{r}}{4\sqrt{2}} - \frac{5r^{3/2}}{32\sqrt{2}} + O(r^{5/2}), \tag{3}$$

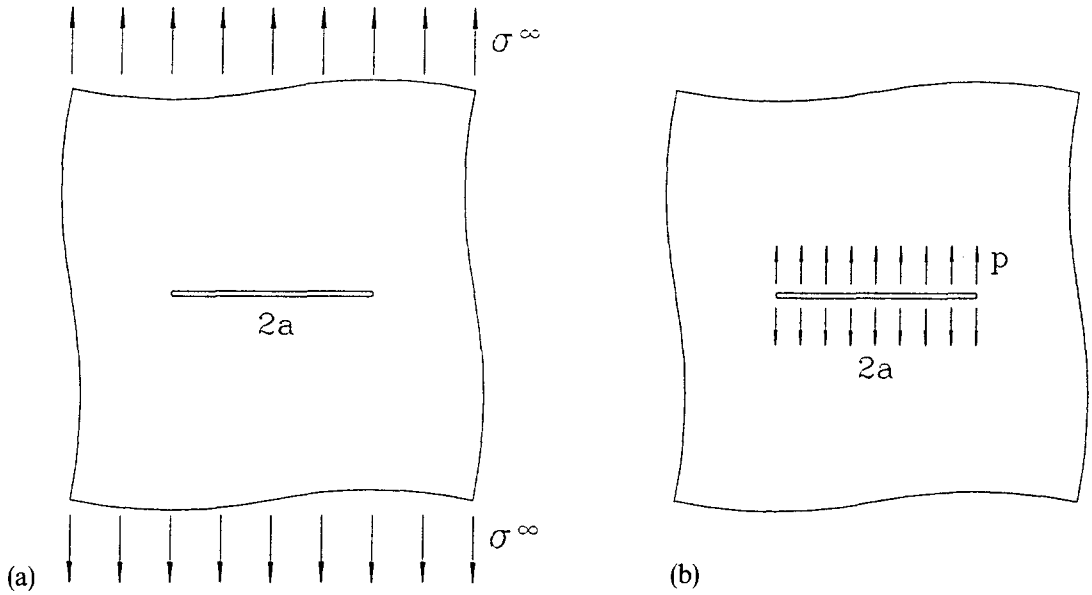

Fig. 1. (a) Griffith crack loaded by remote tension. (b) Pressurized Griffith crack.

$$\sigma_{yy}^{\text{asymptotic-press}} = \frac{1}{\sqrt{2r}} - 1 + \frac{3\sqrt{r}}{4\sqrt{2}} - \frac{5r^{3/2}}{32\sqrt{2}} + O(r^{5/2}). \tag{4}$$

Define the relative error as

$$E \equiv \frac{(\sigma_{yy} - \sigma_{yy}^{\text{asymptotic}})}{\sigma_{yy}}. \tag{5}$$

If the first term is retained in the asymptotic solution, the errors are given by

$$E^{\text{rem}} = \frac{3r}{4} - \frac{23r^2}{32} + O(r^3), \tag{6}$$

$$E^{\text{press}} = -\sqrt{2r} - \frac{5r}{4} - \frac{r^{3/2}}{\sqrt{2}} - \frac{7r^2}{32} + O(r^3). \tag{7}$$

Equations (6) and (7) are plotted in Fig. 2a. It is observed that the region of K-dominance for the pressurized crack is extremely small, as a result of the $\sqrt{r}$-dependence of the relative error.

If terms up to order r^0 are retained in the asymptotic solution (the first term in (3) and the first two terms in (4)), the error for the remote loading is still given by (6), and the error for the pressurized crack becomes

$$E^{\text{press}} = \frac{3r}{4} + \frac{3r^{3/2}}{2\sqrt{2}} + \frac{25r^2}{32} + \frac{7r^{5/2}}{16\sqrt{2}} + O(r^3). \tag{8}$$

Equations (6) and (8) are plotted in Fig. 2b. This figure shows that for the pressure loading the stress field near the crack tip is dominated by the first two terms in the asymptotic expansion.

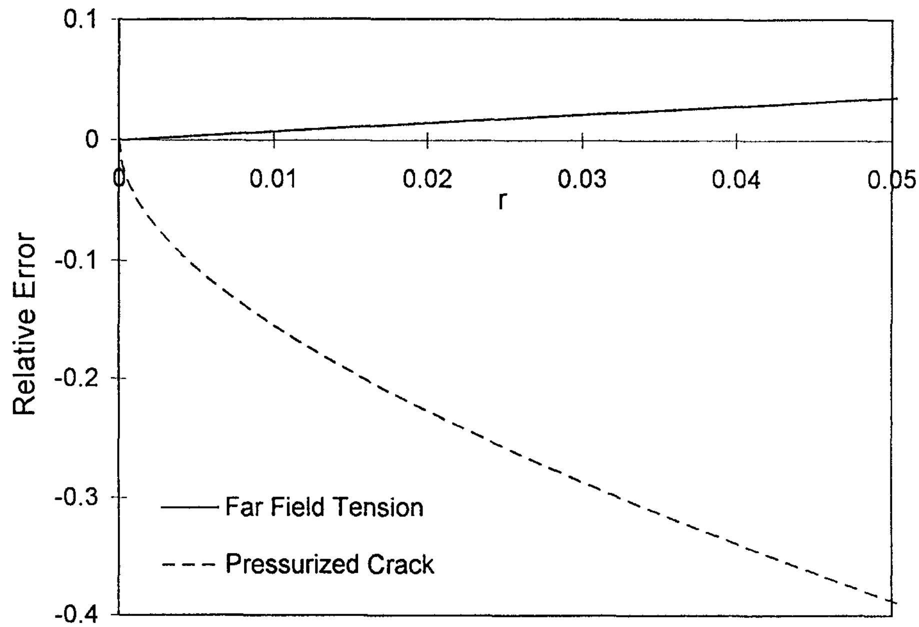

Fig. 2a. Relative error using first term in asymptotic solution.

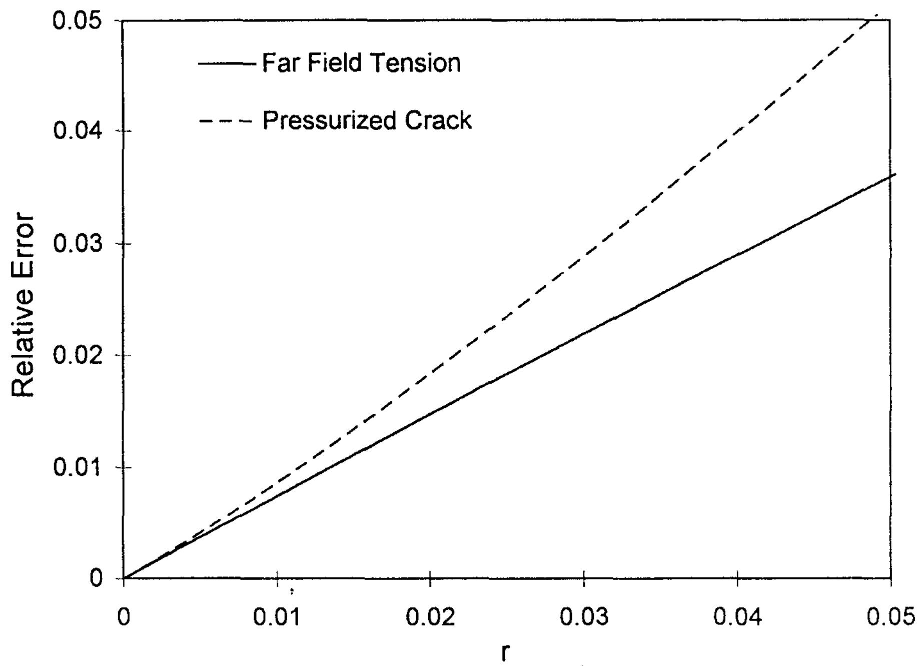

Fig. 2b. Relative error using terms up to order r^0 is asymptotic solution.

Implications

The results suggest that for a pressurized crack a fracture criterion based on critical stress intensity factor may not be applicable for most materials and typical crack lengths, because the region of dominance shown in Fig. 2a will be engulfed by the process zone. Since the stress field for the pressure loading is dominated by the first two terms in the asymptotic expansion, perhaps a two parameter model is necessary.