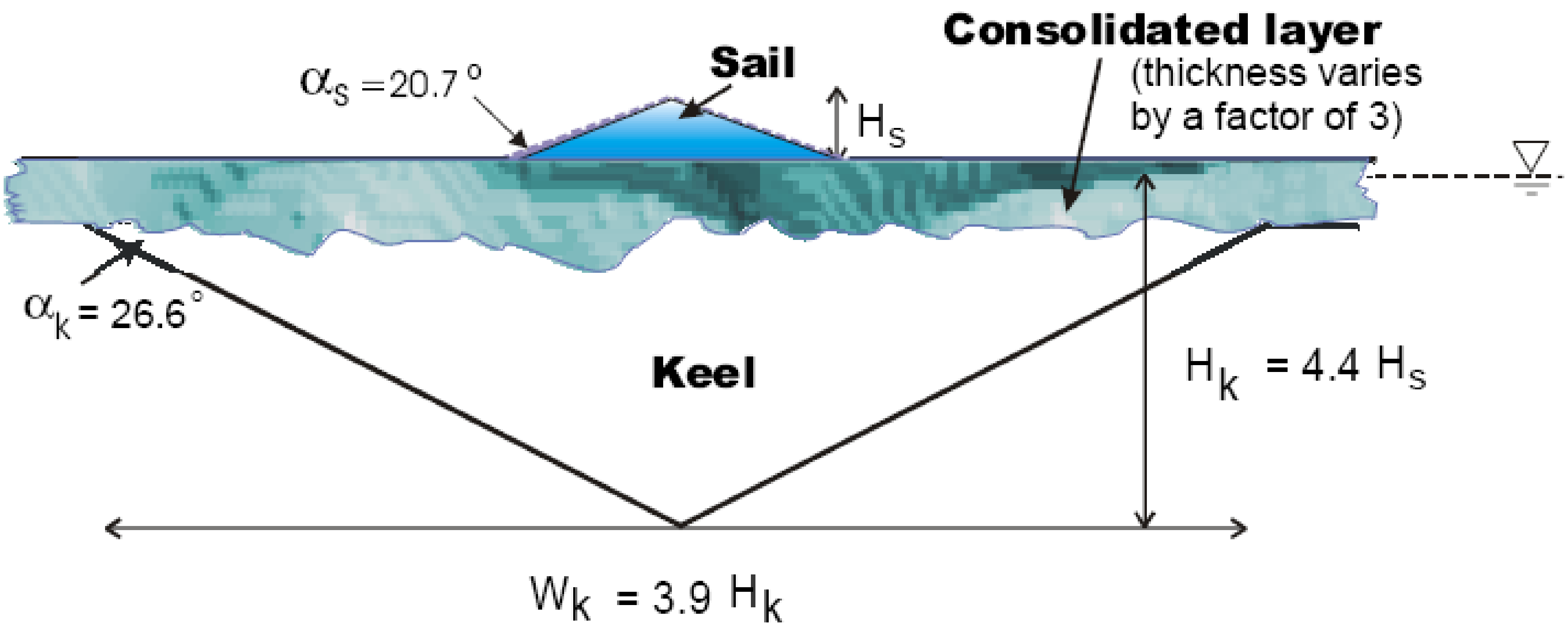
Crack-Tip Parameters in Crystalline Plates with Compliant Grain Boundaries

Yuping Wang & Roberto Ballarini

Sponsors:

NSF, NASA

First-Year Ice Ridges



Total force is addition of sail, keel and consolidated layer contributions

Timco *et al.*, 1999

CONTRIBUTION FROM CONSOLIDATED LAYER

Korzhavin Equation for Ice Force on a Vertical-Sided Structure
(Assumes ice crushing; based on ice indentation and derived for bridge piers)

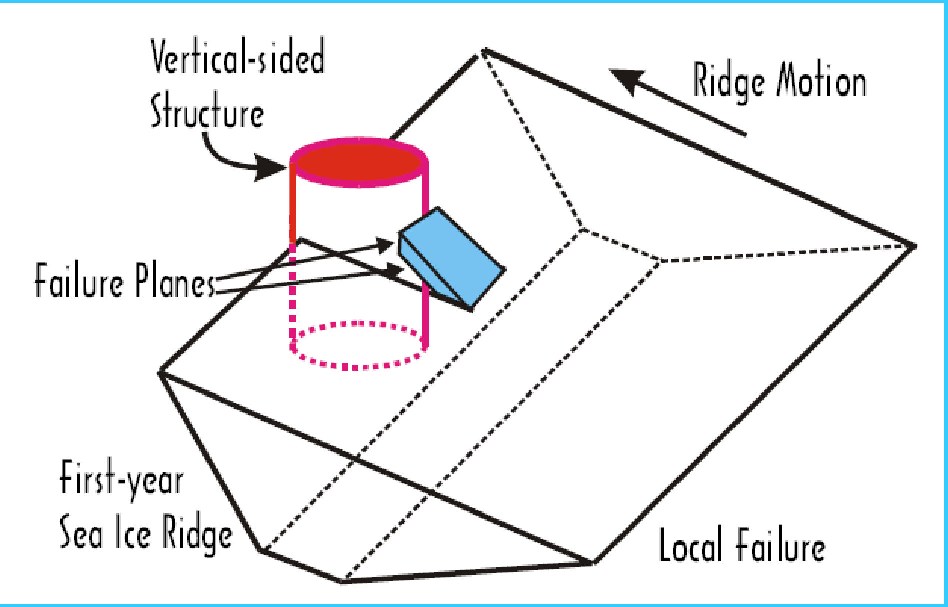
$$F = pDh$$

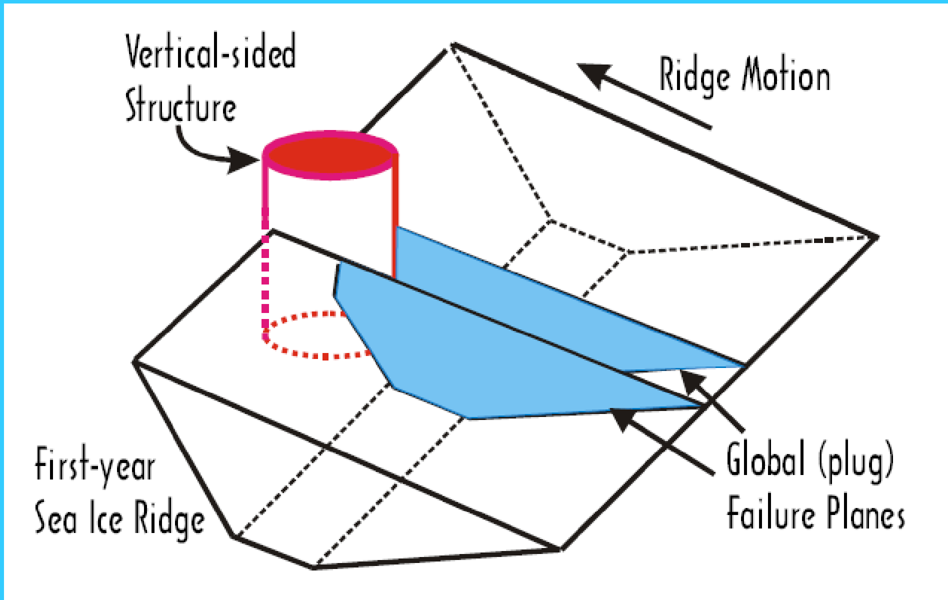
- p: ice pressure
- D: structure width
- h: ice thickness

$$p = Ikm\sigma_c$$

- I: indentation factor, ~1.2-4.5; F(D/h) and ice anisotropy
- k: contact factor, 0.3-1.0; low for cold ice high for war ice
- m: shape factor, 0.9 (1.0) round (flat) structure
- σ_c: uniaxial compressive strength

CONTRIBUTIONS FROM SAIL AND KEEL
Models borrowed from concrete/geomaterials plasticity models



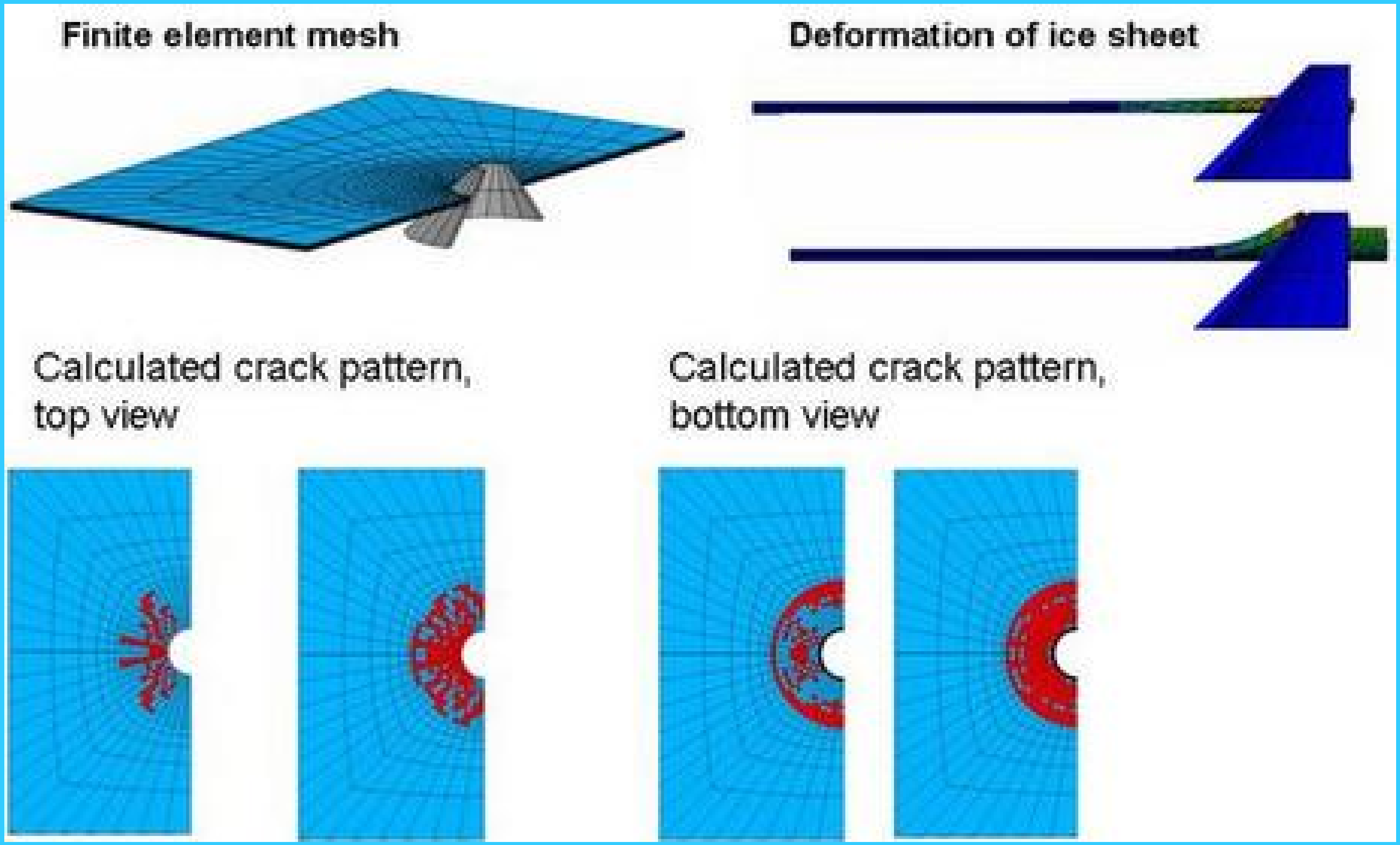


Local Failure Model

Global Failure Model

	Predicted Component Loads (MN)				
	Ridge 1 (MN)	Ridge 2 (MN)	Ridge 3 (MN)	Ridge 4 (MN)	Ridge 5 (MN)
Consolidated Layer					
Korzhavin (maximum)	1,010	1,010	1,820	1,820	1,820
Korzhavin (minimum)	20	20	36	36	36
Korzhavin (best guess)	58	58	105	105	105
Local Failure					
Dolgopolov (1975), q=1	8	13	8	8	8
Prodanovic (1979), keel	3	4	3	3	3
Mellor (1980), sail	4	7	4	4	4
Mellor (1980), keel	8	13	8	8	8
Mellor (1980), sail+keel	12	20	12	12	12
Hoikkanen (1984), sail	3	6	3	3	3
Hoikkanen (1984), keel	14	23	14	14	14
Hoikkanen (1984), sail+keel	17	29	17	17	17
Croasdale (1994), wedge	3	5	3	3	3
Weaver (1994), friction	7	12	7	7	7
Weaver (1994), cohesion	6	6	6	6	6
Global Failure					
Croasdale (1980), keel	0.5	0.8	0.5	0.5	0.5
Prodanovic (1981), keel	0.1	0.2	0.1	0.1	0.1
Croasdale (1993)	3.3	4.5	3.3	3.3	3.3

	Predicted Component Loads (MN)				
	Ridge 1 (MN)	Ridge 2 (MN)	Ridge 3 (MN)	Ridge 4 (MN)	Ridge 5 (MN)
Full-Scale Measured Load	**45**	**70**	**60**	**65**	**70**
Consolidated Layer + Local Failures					
Dolgopolov (1975)	66	72	113	113	113
Prodanovic (1979)	61	62	108	108	108
Mellor (1980)	70	78	117	117	117
Hoikkanen (1984)	76	87	122	122	122
Croasdale (1994)	61	63	108	108	108
Weaver (1994c)	65	71	112	112	112
Weaver (1994f)	64	65	111	111	111
Consolidated Layer + Global Failure					
Croasdale (1980)	59	59	105	105	105
Prodanovic (1981)	58	58	105	105	105
Croasdale (1993)	62	63	108	108	108

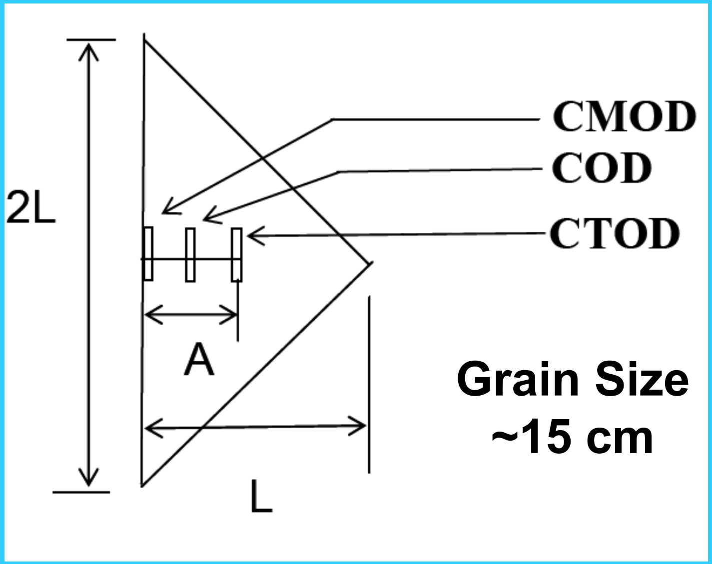


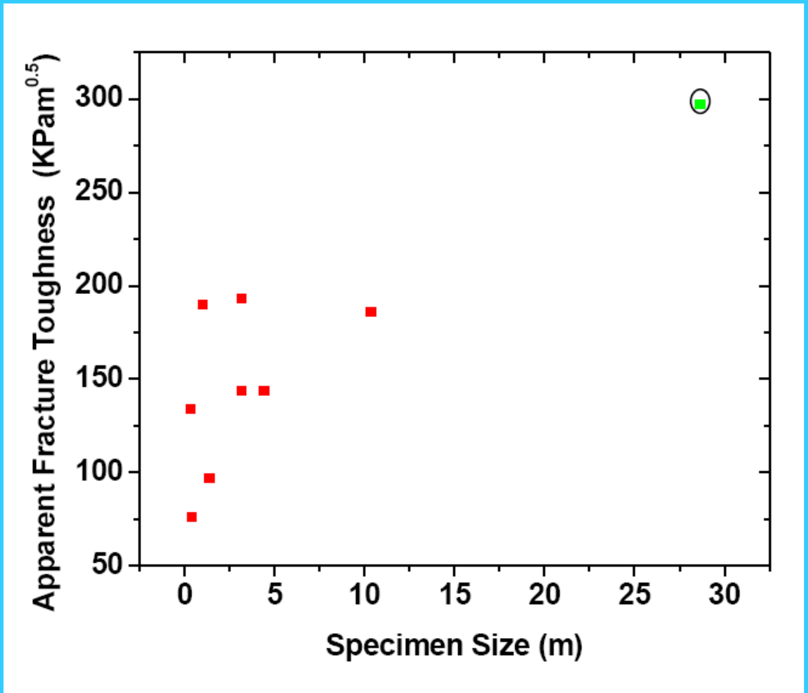
Simulation of Ice Sheet Break-up

http://en.tek.norut.no/layout/set/print/content/view/full/10706

MOTIVATION
Do Experiments on Warm Lake Ice Exhibit Size Effects?

J. Dempsey *et al.*, IJF 1995

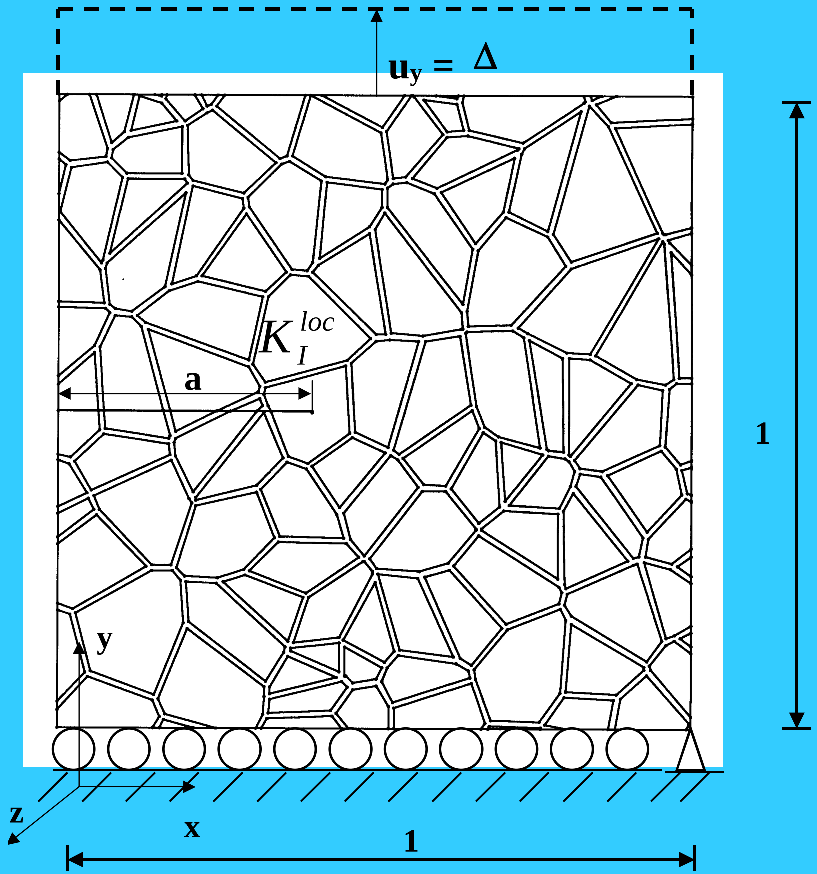




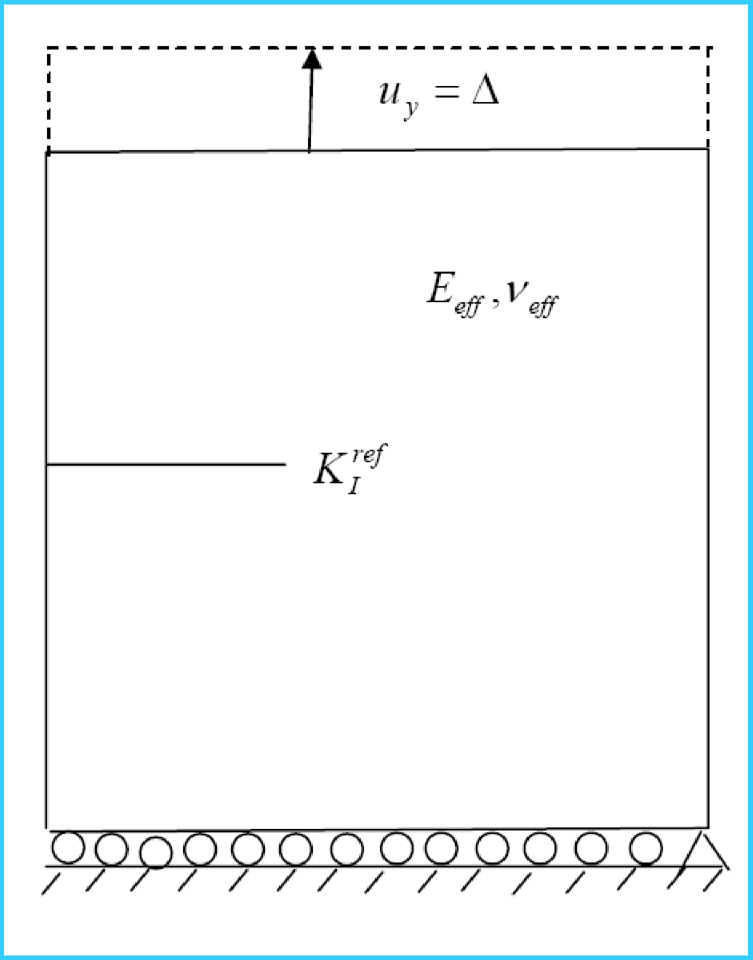
Test#	Size (m)	E_{COD} (GPA)	E_{CMOD} (GPA)
1	0.34	---	---
2	0.41	---	5.7
3	1.04	2.8	---
4	1.41	3.5	4.3
5	3.18	---	2.9
6	3.20	3.6	8.0
7	4.42	5.7	7.6
8	10.36	7.7	3.2
9	28.64	10.0	---

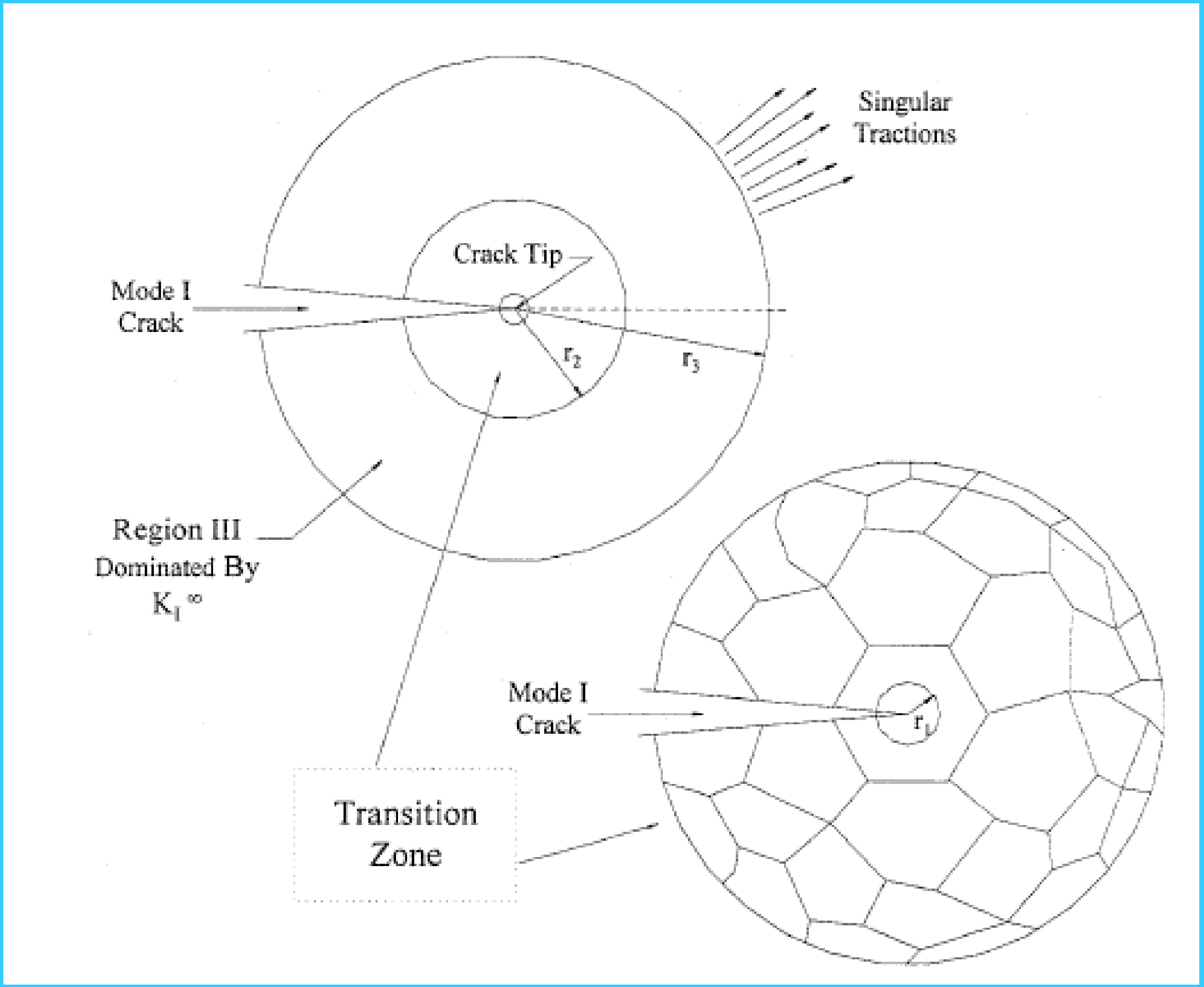
Scatter also found in short-time elastic modulus calculated from crack openings at various points

Discrete Structure



Homogenized Structure

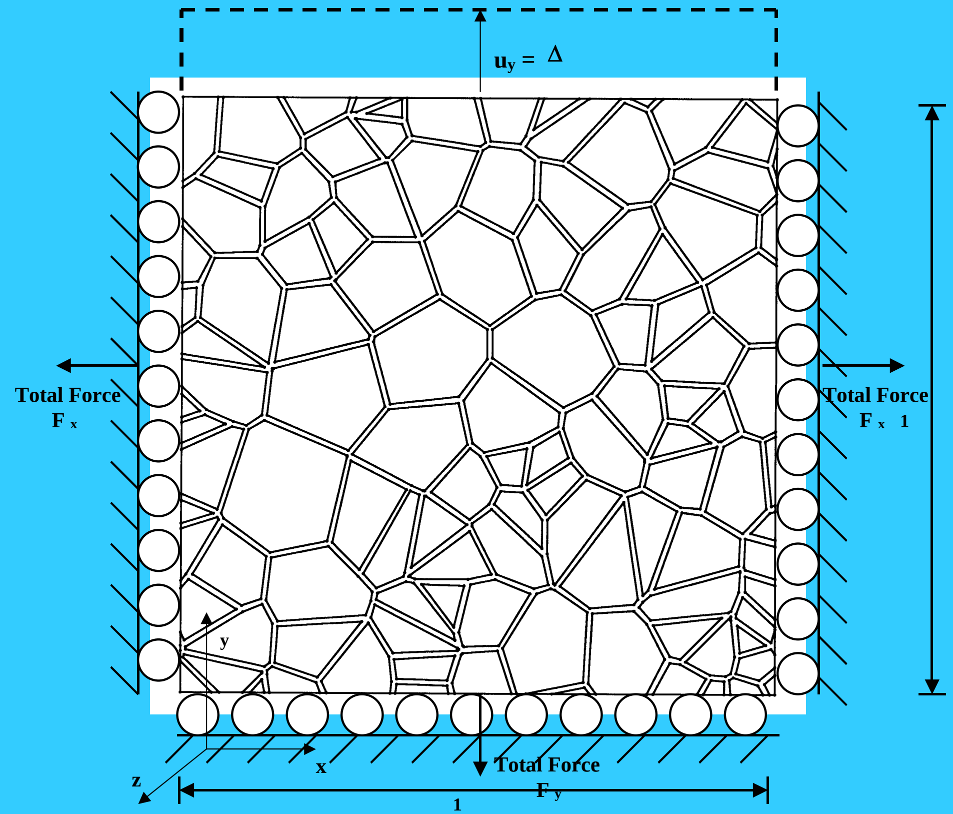




Grain Brittleness, GB≡d/r_p

$$r_p \approx \left(K_c/\sigma_u\right)^2$$

Effective Elastic Moduli of Aggregates

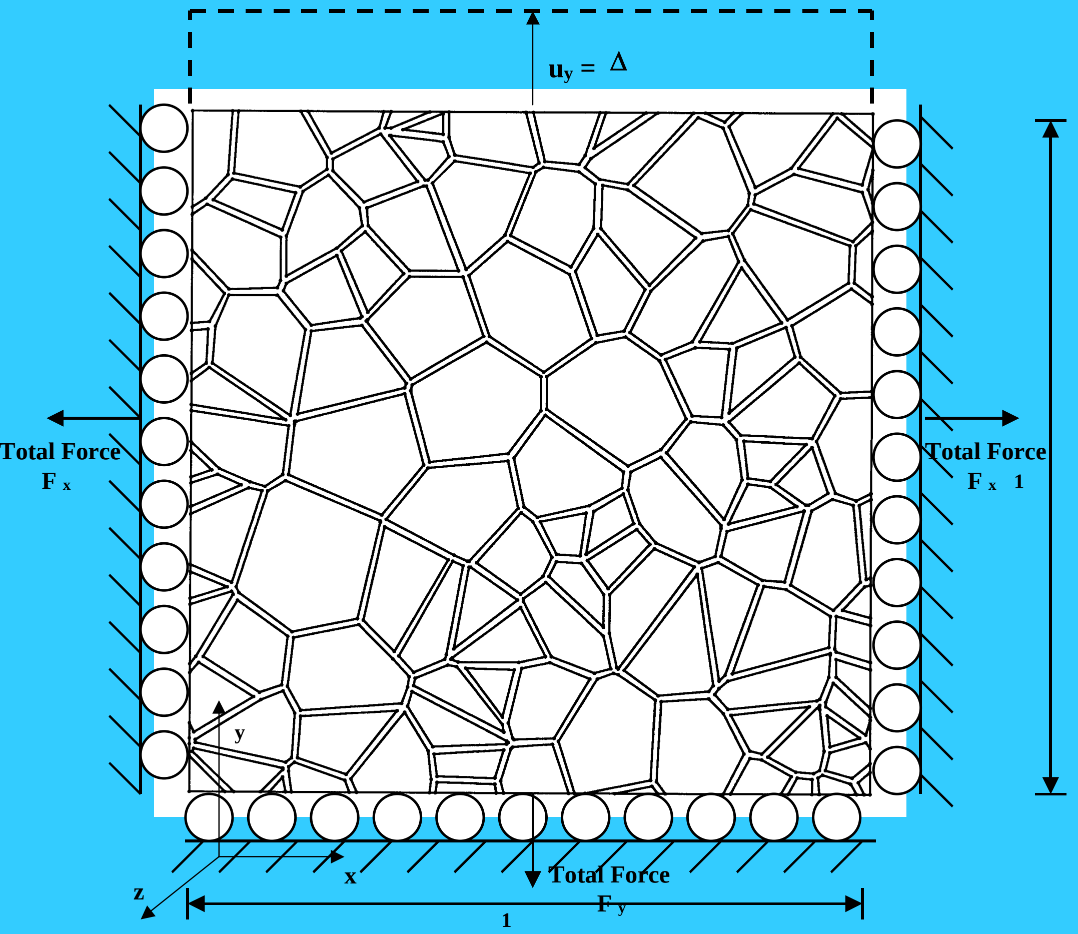


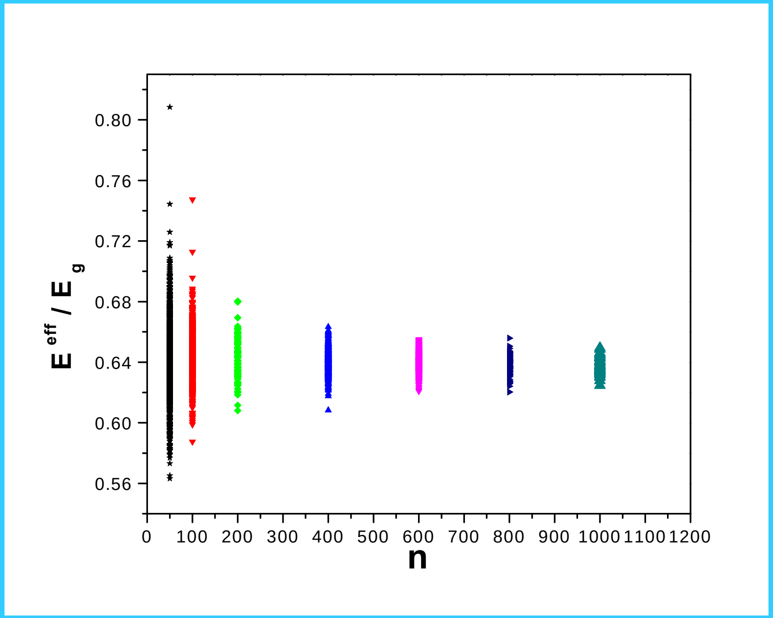
$$\begin{Bmatrix} \sigma^i_{xx} = F^i_x / t \\ \sigma^i_{yy} = F^i_y / t \\ \sigma^i_{xy} = 0 \end{Bmatrix} = \frac{E^i}{(1+\nu^i)(1-2\nu^i)} \begin{bmatrix} 1-\nu^i & \nu^i & \\ \nu^i & 1-\nu^i & \\ & & \frac{1}{2}(1-2\nu^i) \end{bmatrix} \begin{Bmatrix} \varepsilon^i_{xx} = 0 \\ \varepsilon^i_{yy} = \Delta \\ 2\varepsilon^i_{xy} = 0 \end{Bmatrix}$$

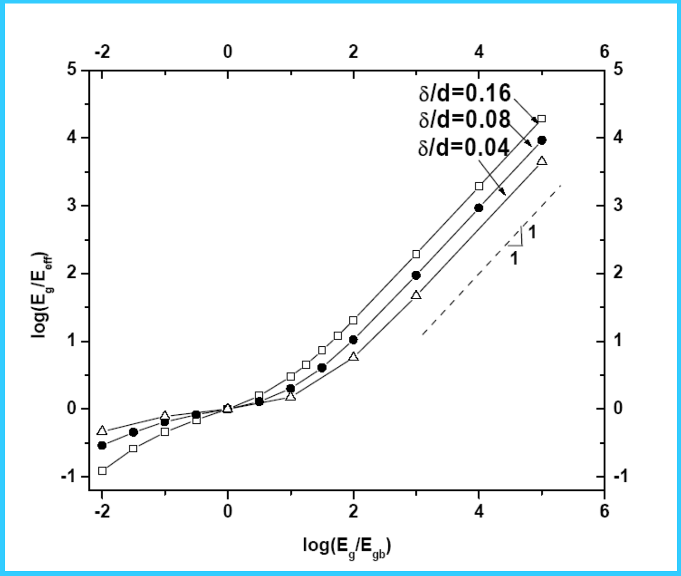
$$E^i = \frac{(1+\nu^i)(1-2\nu^i)}{\nu^i} \frac{F^i_x}{t\Delta} \qquad \nu^i = \frac{1}{1+F^i_y / F^i_x}$$

$$E_{eff} = \langle E^i \rangle \qquad \nu_{eff} = \langle \nu^i \rangle$$

Effective Elastic Moduli of Aggregates

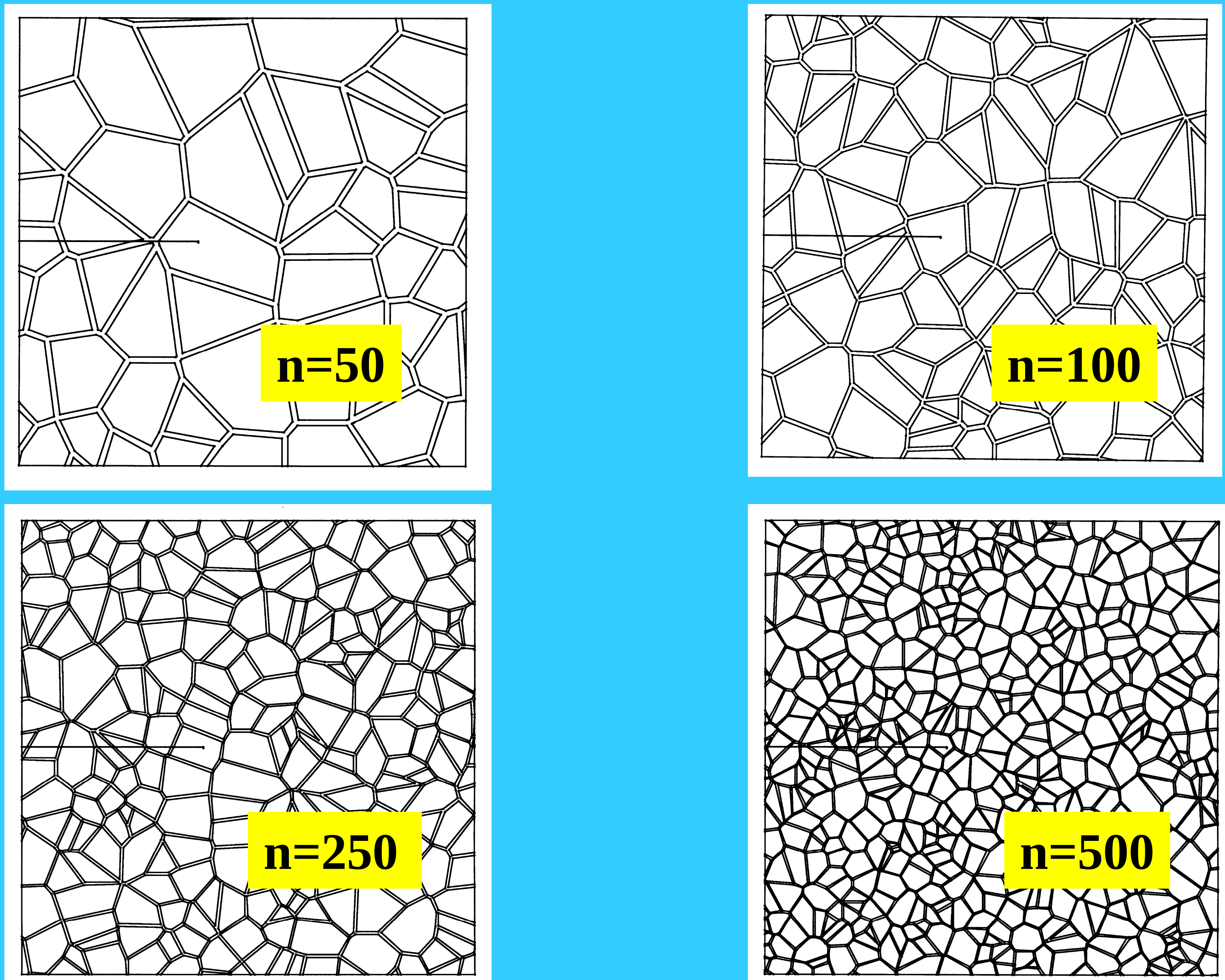




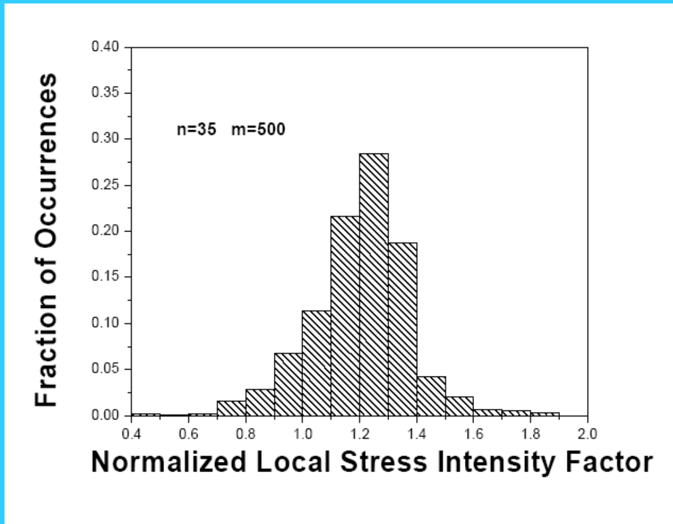


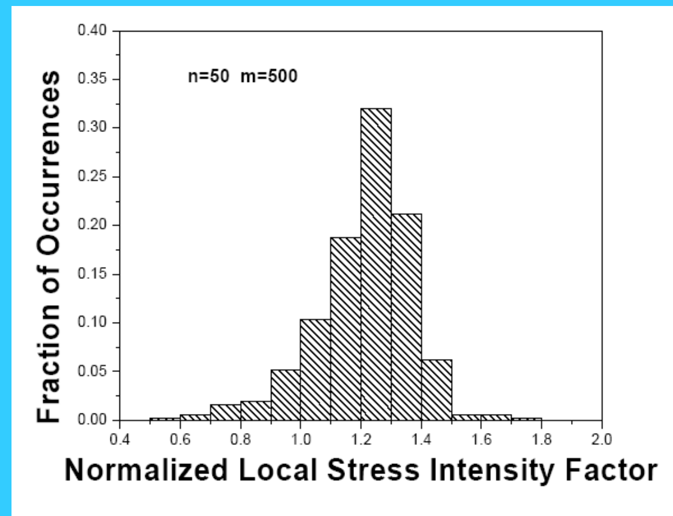
$$\frac{E_g}{E_{gb}} >> 1 \quad , \quad E_{eff} \sim C_1(\delta/d)E_{gb}$$

Effects of Number of Grains

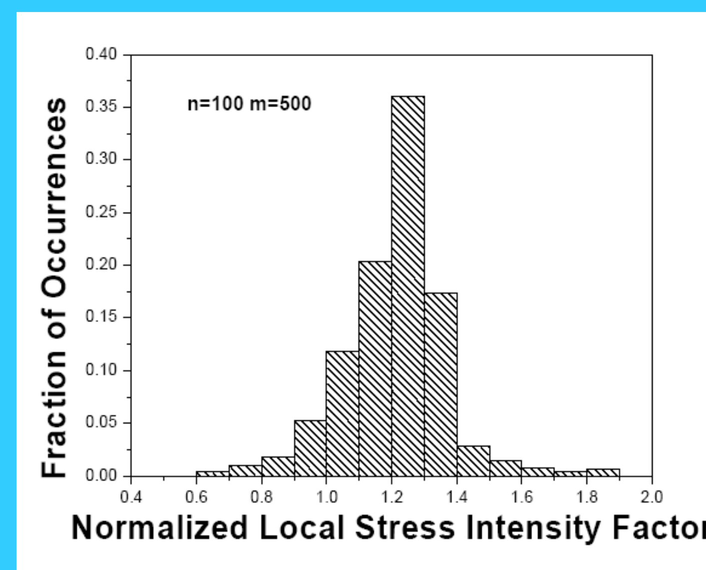


$$K_I^{loc} / K_I^{ref}$$





$$E_g / E_{eff} \approx 1.55 \ , \ \nu_{eff} \approx 0.49$$

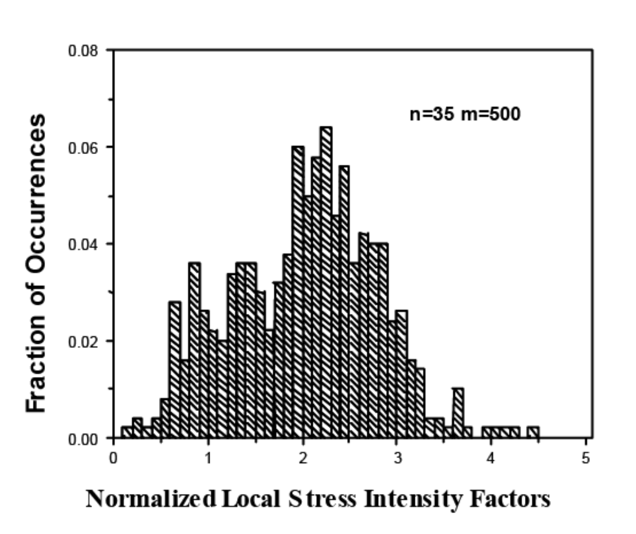


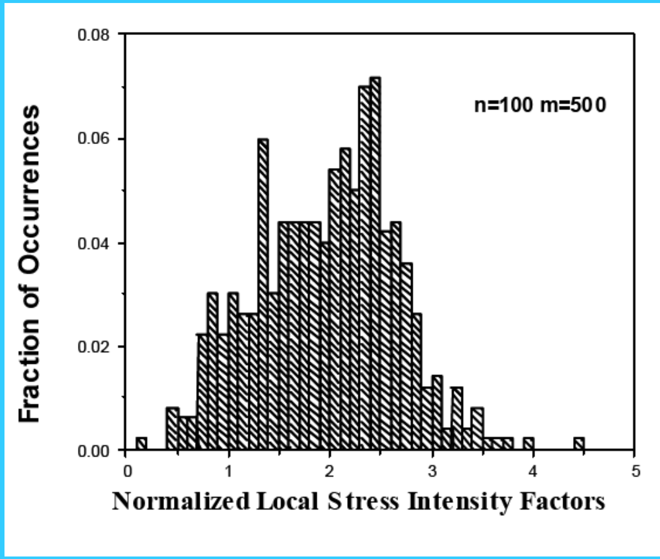
$$\nu_g = 0.49$$

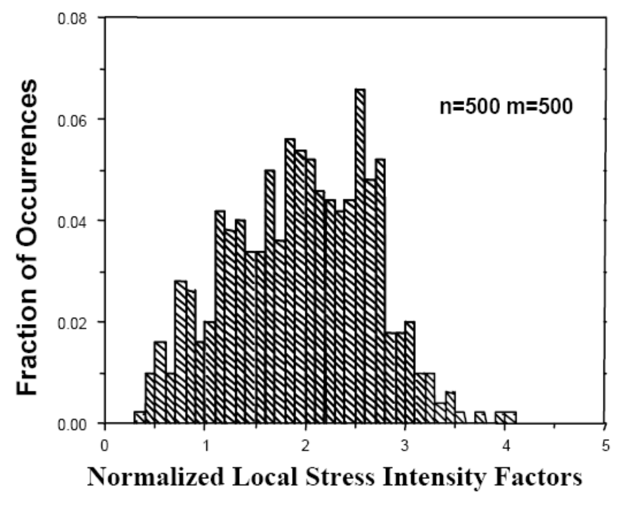
$$\nu_{gb} = 0.499$$

$$\mu_{gb} / \mu_g = 0.1$$

$$K_I^{loc} / K_I^{ref}$$







$$E_g / E_{eff} \approx 6.06, \ \nu_{eff} \approx 0.41$$

$$\nu_g = 0.2$$

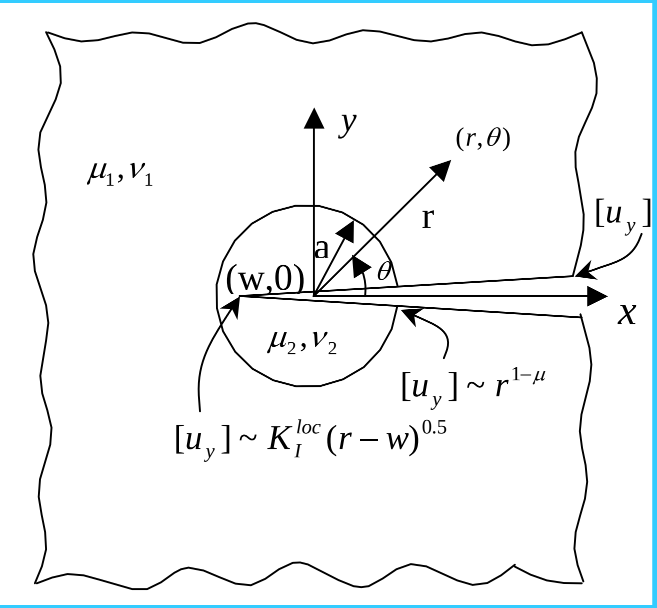
$$\nu_{gb} = 0.49$$

$$\mu_{gb} / \mu_g = 0.004$$

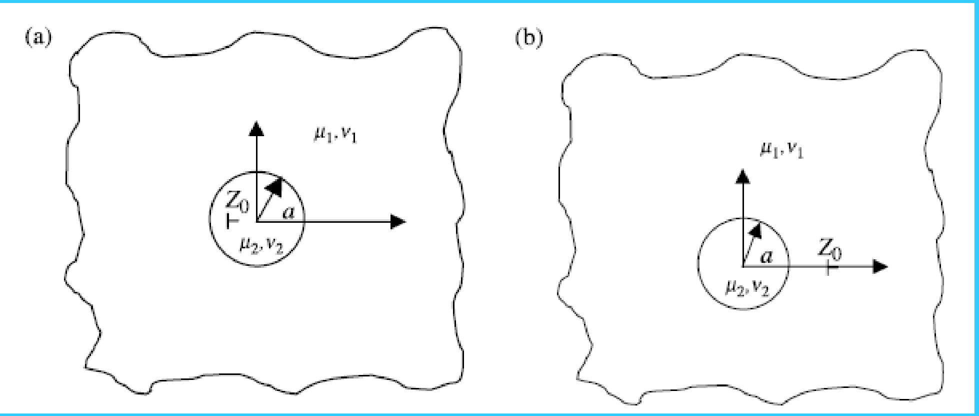
Very Weak Dependence on Number of Grains

E_g / E_{eff}	ν_g	ν_{eff}	n	m	K_{avg}	K_{sd}
1.55	0.49	0.49	approximate		1.24	0.06
			35	500	1.22	0.18
			50	500	1.21	0.17
			100	500	1.21	0.16
			500	500	1.23	0.17
1.47	0.2	0.34	approximate		1.15	0.03
			35	447	1.13	0.13
			100	280	1.13	0.11
			500	126	1.13	0.11
6.06	0.2	0.41	approximate		2.20	0.60
			9	300	2.01	0.83
			35	500	2.02	0.77
			100	500	1.96	0.68
			500	500	1.95	0.71
2.00	0.3	0.28	approximate		1.48	0.16
			50	500	1.42	0.18
			500	500	1.41	0.18
10.53	0.3	0.24	approximate		3.34	0.97
			50	500	3.02	1.00
			500	395	2.91	0.97
0.65	0.3	0.31	approximate		0.77	0.05
			50	500	0.73	0.11
			500	500	0.73	0.11

Approximate analytical model



$$[u_y] = \frac{K_I^{\infty}(\kappa_1 + 1)}{\mu_1} \sqrt{\frac{r}{2\pi}}$$



Green's Functions

$$\sigma_{rr} + i\tau_{r\theta} = \Phi_i(z) + \overline{\Phi_i(z)} - \frac{\bar{z}}{z}[z\overline{\Phi_i'(z)} + \overline{\Psi_i(z)}]$$

$$\sigma_{\theta\theta} + i\tau_{r\theta} = \Phi_i(z) + \overline{\Phi_i(z)} + \frac{z}{\bar{z}}[\bar{z}\Phi_i'(z) + \Psi_i(z)]$$

$$\frac{\partial}{\partial\theta}(u_x + iu_y) = \frac{iz}{2\mu_i}\left\{\kappa_i\Phi_i(z) - \overline{\Phi_i(z)} + \frac{\bar{z}}{z}[z\overline{\Phi_i'(z)} + \overline{\Psi_i(z)}]\right\}$$

$$\begin{aligned}\Phi_1(z) \; &= \; \frac{A}{z-z_0} - \frac{\beta-\alpha}{1+\beta}\left\{\frac{\bar{A}}{a^2/z-\bar{z}_0} + \frac{a^2}{z}\left[\frac{\bar{A}}{(a^2/z-\bar{z}_0)^2}\right] - \right. \\ &\left. - \frac{a^2}{z^2}\left[\frac{\bar{B}}{a^2/z-\bar{z}_0} + \frac{z_0\bar{A}}{(a^2/z-\bar{z}_0)^2}\right] + \frac{\bar{A}}{\bar{z}_0}\right\}\end{aligned}$$

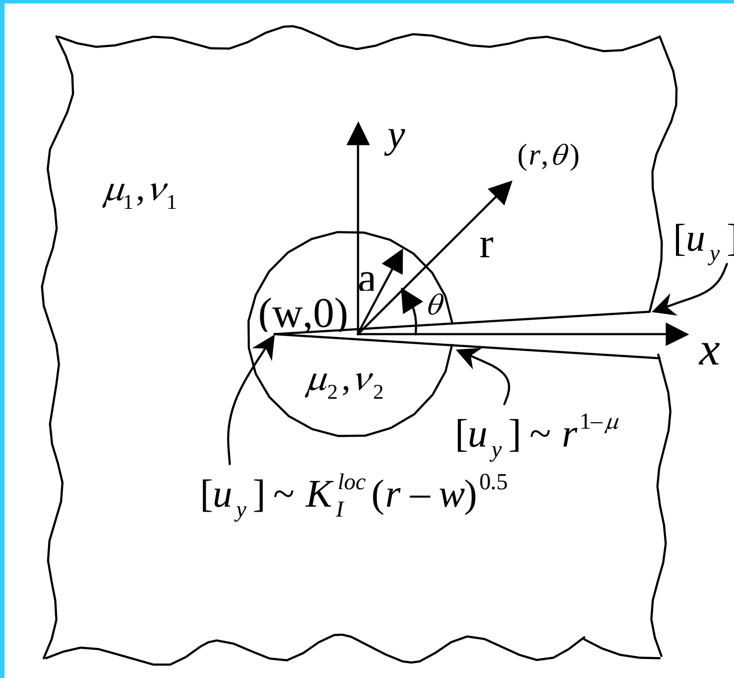
$$\begin{aligned}\Psi_1(z) \; &= \; \frac{B}{z-z_0} + \frac{A\bar{z}_0}{(z-z_0)^2} + \frac{a^2}{z^2}\left\{\Phi_1(z) - z\Phi_1'(z) + \left(\frac{\alpha+\beta}{\beta-1}\right)\frac{\bar{A}}{a^2/z-\bar{z}_0} - \right. \\ &\left. - \frac{A}{z-z_0} - \frac{zA}{(z-z_0)^2} - \frac{A}{z_0} + \left(\frac{1-\alpha}{1-\beta}\right)\bar{C}\right\}\end{aligned}$$

$$\Phi_2(z) = \left(\frac{1+\alpha}{1-\beta}\right)\frac{A}{z-z_0} + \left(\frac{\alpha-\beta}{1-\beta}\right)C$$

$$\begin{aligned}\Psi_2(z) \; &= \; \left(\frac{1+\alpha}{1+\beta}\right)\left[\frac{B}{z-z_0} + \frac{A\bar{z}_0}{(z-z_0)^2}\right] + \\ &+ \frac{a^2}{z^2}\left\{\Phi_2(z) - z\Phi_2'(z) - \left(\frac{1+\alpha}{1+\beta}\right)\left[\frac{A}{z-z_0} + \frac{zA}{(z-z_0)^2} + \frac{A}{z_0}\right] + \bar{C}\right\}\end{aligned}$$

Dislocation outside

Approximate analytical model



$$[u_y] = \frac{K_I^{\infty}(\kappa_1 + 1)}{\mu_1}\sqrt{\frac{r}{2\pi}}$$

$$\int_a^{\infty}\frac{2b_1(t)}{x-t}dt + \int_a^{\infty}K_{11}(x,t)b_1(t)dt + \int_w^{a}K_{12}(x,t)b_2(t)dt = 0 \qquad x > a$$

$$\int_w^{a}\frac{2b_2(t)}{x-t}dt + \int_w^{a}K_{21}(x,t)b_2(t)dt + \int_a^{\infty}K_{22}(x,t)b_1(t)dt = 0 \qquad w < x < a$$

$$\int_{-1}^{1}\frac{\overline{b_i}(\eta)}{\xi-\eta}d\eta \approx \sum_{k=1}^{N}W_k\frac{\overline{g}_i(\eta_k)}{\xi_j-\eta_k} \qquad \int_{-1}^{1}K_{mn}(\eta,\xi)\overline{b}_i(\eta)d\eta \approx \sum_{k=1}^{N}W_kK_{mn}(\xi_j,\eta_k)\overline{g}_i(\eta_k)$$

$$W_k = -\frac{2N+s_1+s_2+2}{(N+1)!(N+s_1+s_2+1)}\frac{\Gamma(N+s_1+1)\Gamma(N+s_2+1)}{\Gamma(N+s_1+s_2+1)}\times\frac{2^{s_1+s_2}}{P_N^{'(s_1,s_2)}(\eta_k)P_{N+1}^{(s_1,s_2)}(\eta_k)}$$

Meccanica, 2003

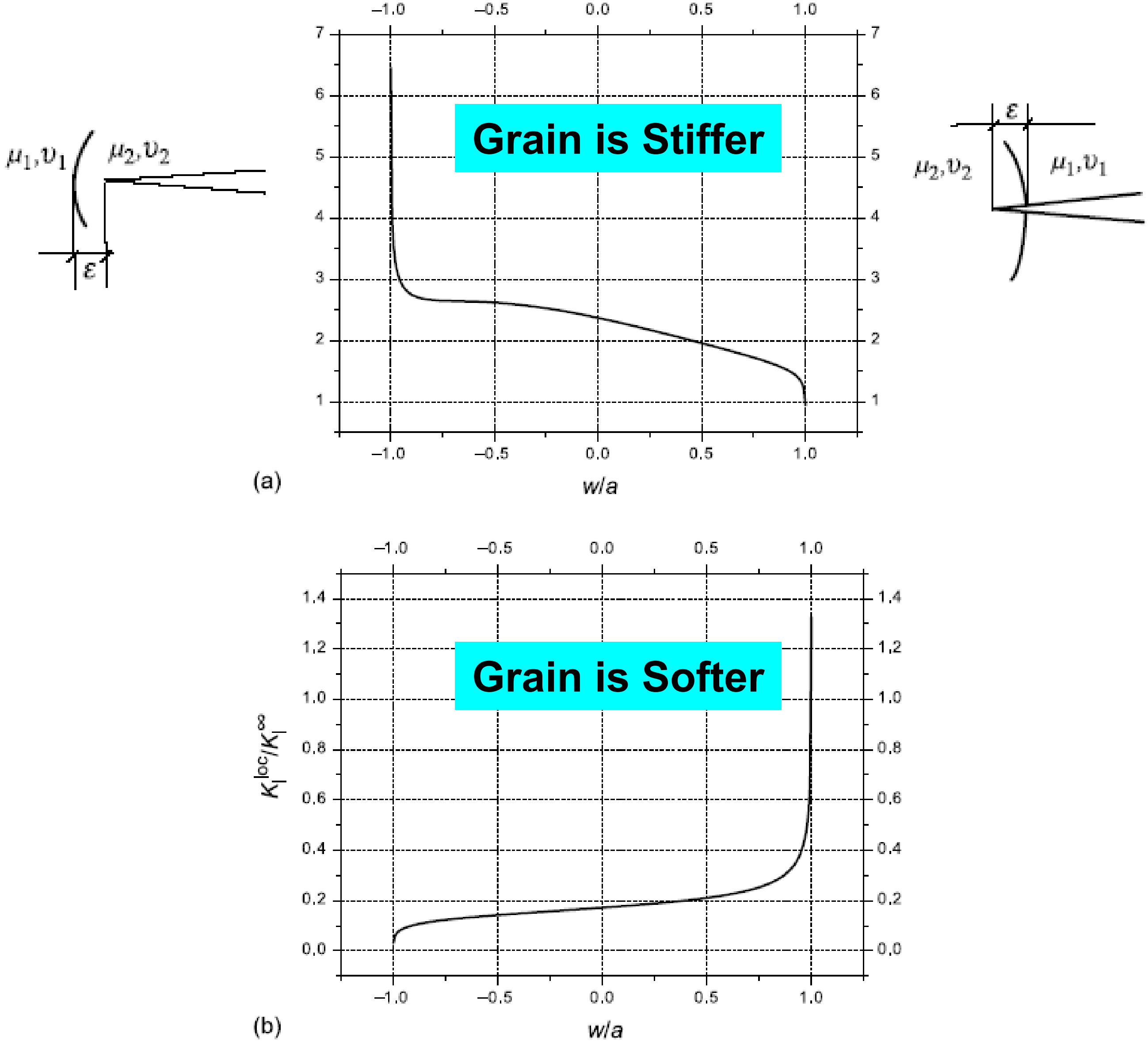


Figure 7. Variation of normalized microscopic stress intensity factor as the crack traverses the inhomogeneity for the parameters (a) $\mu_1/\mu_2 = 0.14$, $\kappa_1 = 1.40$, $\kappa_2 = 2.2$ ($\alpha = 0.6812$, $\beta = 0.0735$) and (b) $\mu_1/\mu_2 = 10$, $\kappa_1 = \kappa_2 = 2.6$ ($\alpha = -0.8182$, $\beta = -0.3636$).

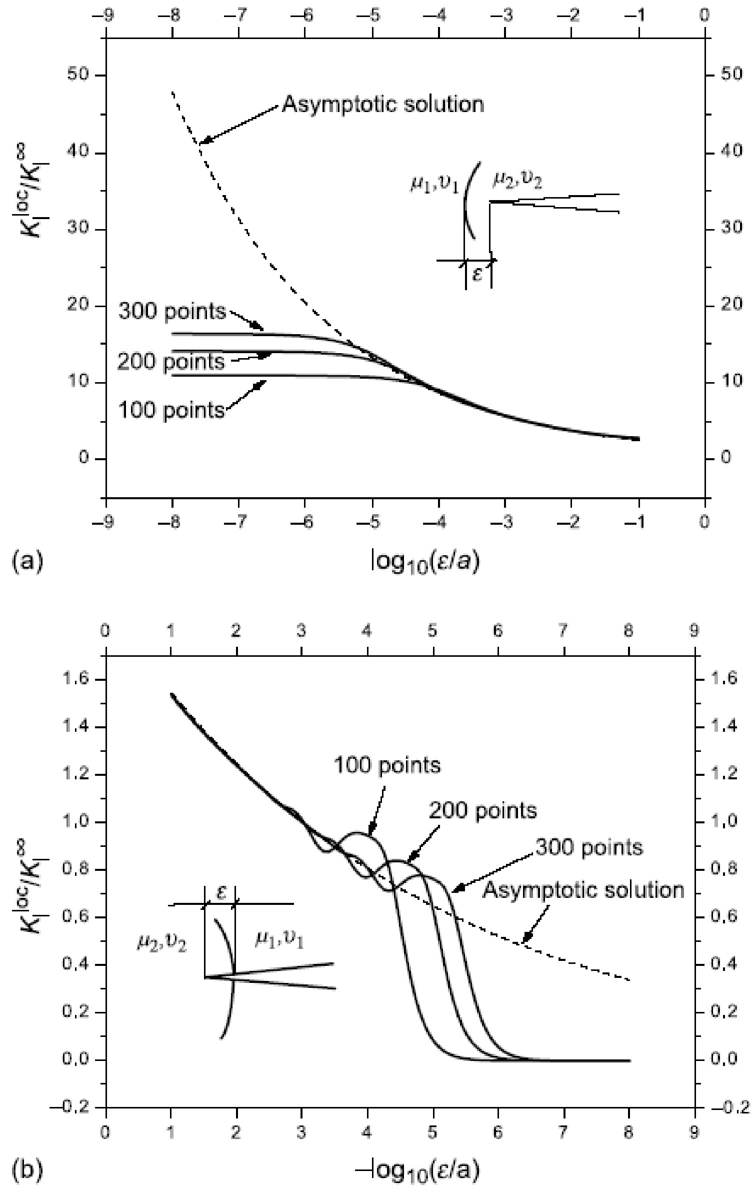


Figure 8. Variation of normalized microscopic stress intensity factor associated with $\mu_1/\mu_2 = 0.14$, $\kappa_1 = 1.40$, $\kappa_2 = 2.2$ ($\alpha = 0.6812$, $\beta = 0.0735$) and the crack tip being very close to (a) the left interface and (b) the right interface. Note that if the crack tip is close to the left interface, $\varepsilon/a = (w + a)/a$, while if it is close to the right interface, $\varepsilon/a = (a - w)/a$.

Very Weak Dependence on Number of Grains

E_g / E_{eff}	ν_g	ν_{eff}	n	m	K_{avg}	K_{sd}
1.55	0.49	0.49	approximate		1.24	0.06
			35	500	1.22	0.18
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			100	500	1.21	0.16
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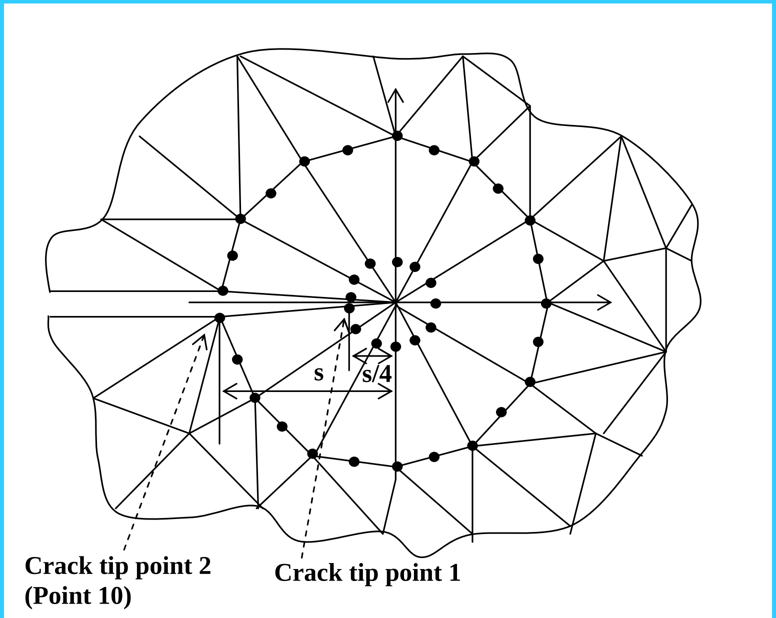
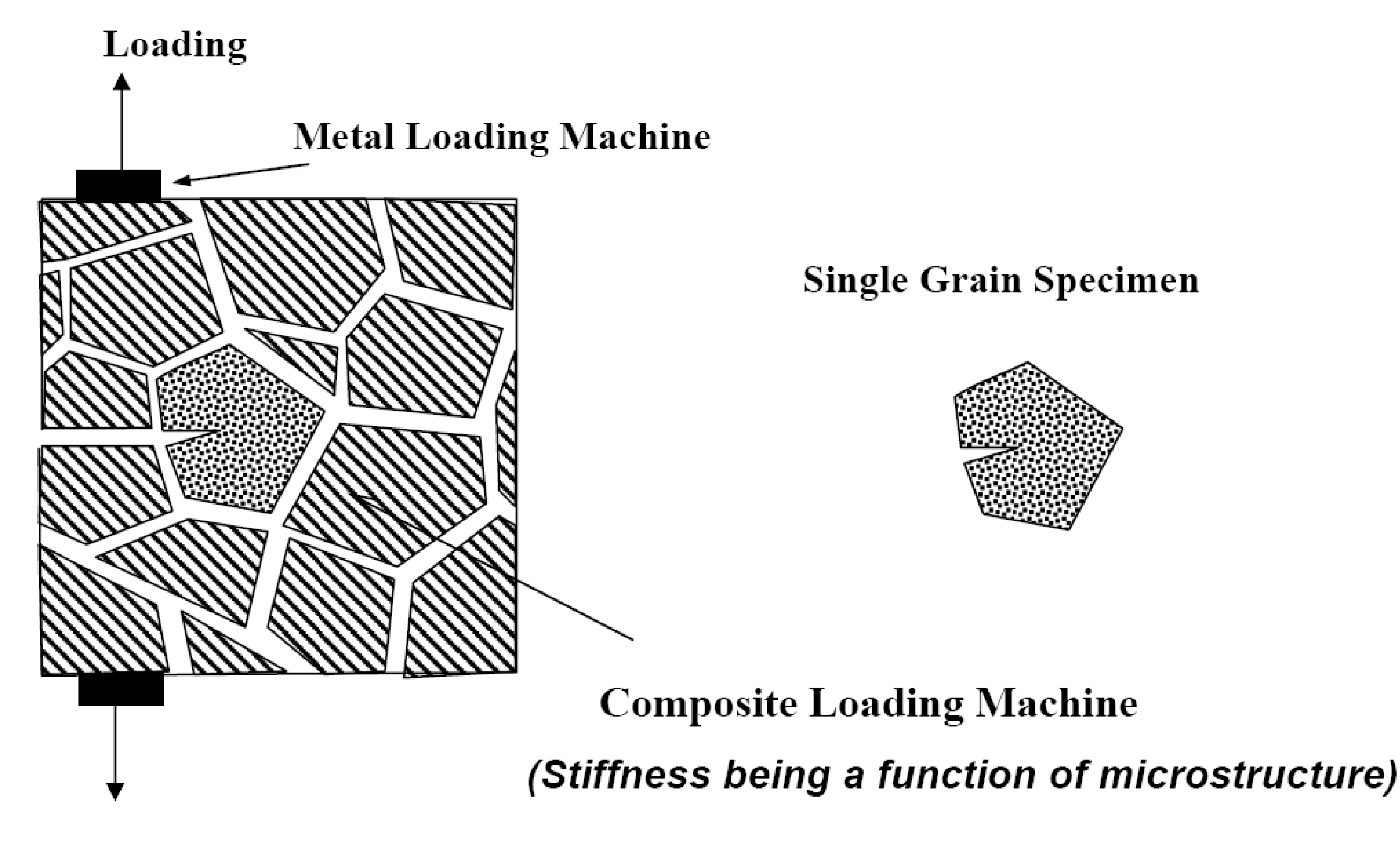
6.06	0.2	0.41	approximate		2.20	0.60
			9	300	2.01	0.83
			35	500	2.02	0.77
			100	500	1.96	0.68
			500	500	1.95	0.71
2.00	0.3	0.28	approximate		1.48	0.16
			50	500	1.42	0.18
			500	500	1.41	0.18
10.53	0.3	0.24	approximate		3.34	0.97
			50	500	3.02	1.00
			500	395	2.91	0.97
0.65	0.3	0.31	approximate		0.77	0.05
			50	500	0.73	0.11
			500	500	0.73	0.11



$\frac{\Delta_i}{(F_y/E_{eff})}$		Point Label	mean	Standard deviation	C.O.V.	minimum	maximum
n=50	CMOD	1	0.51	0.047	0.092	0.35	0.63
		2	0.47	0.044	0.094	0.32	0.58
		3	0.43	0.040	0.094	0.29	0.54
		4	0.39	0.039	0.099	0.24	0.50
	COD	5	0.35	0.035	0.10	0.20	0.46
		6	0.30	0.032	0.10	0.20	0.38
		7	0.26	0.030	0.12	0.16	0.33
		8	0.20	0.027	0.13	0.13	0.27
		9	0.13	0.025	0.20	0.067	0.20
	CTOD 2	10	0.0013	2.1E-04	0.17	6.8E-04	0.002
	CTOD 1		6.4E-04	1.1E-04	0.17	3.4E-04	9.7E-04


Loading
Metal Loading Machine
Single Grain Specimen
Composite Loading Machine
(Stiffness being a function of microstructure)

CONCLUSIONS