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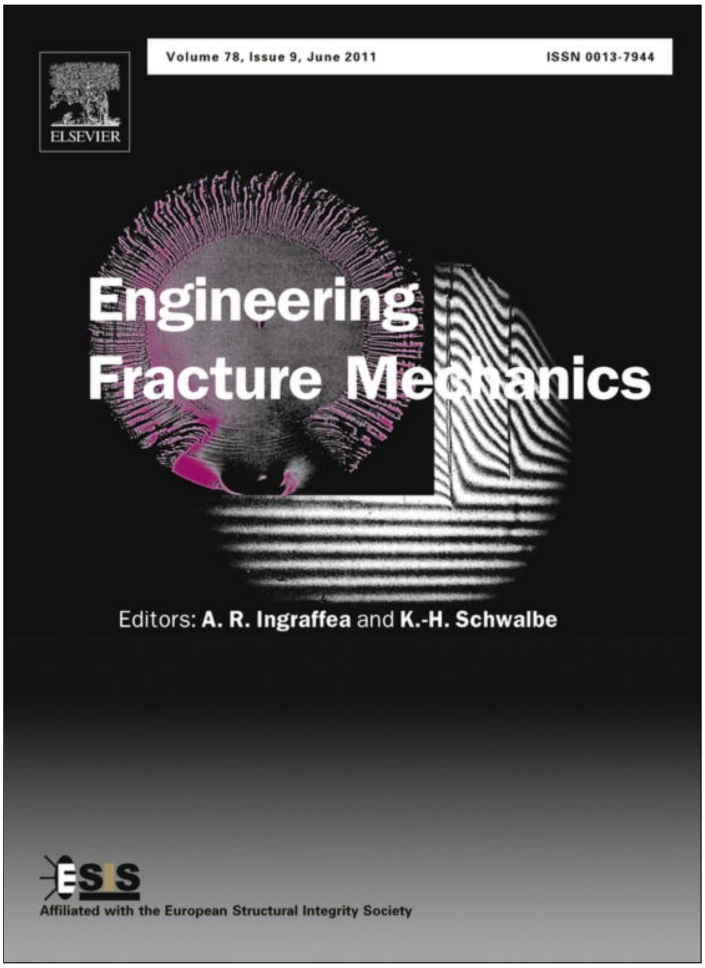

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Distributed damage creates flaw tolerance

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ABSTRACT

A qualitative micromechanical fracture mechanics model is presented that shows how a structure that is sensitive to the presence of a single crack or hole can be rendered flaw tolerant by the presence of an interacting distribution of such flaws. The simple model was inspired by the ductile fracture experienced by the under-designed gusset plates recovered from the I-35W Bridge collapse and by the experimentally measured increase in toughness of concrete damaged by fire.

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1. Introduction

This paper is concerned about the toughness of structures containing distributed and interacting flaws (cracks and holes for example) that fail as a result of the propagation of thin bands, wherein irreversible deformation occurs such as microcracking and aggregate interlock in concrete or plasticity in metals. Specific attention is paid to whether the failure is brittle or ductile, an important issue ever since Griffith [1] gave birth to the field of fracture mechanics and revisited recently [2,3]. A simple illustrative micromechanics model is used to demonstrate how a structure that is sensitive to the presence of an isolated flaw can be rendered flaw tolerant by the introduction of closely spaced flaws.

2. Model

Before proceeding to the model, a brief review is provided of the coupling between the characteristic dimensions of a structural component and the properties of its constituent material. This is arguably the most useful concept provided by fracture mechanics theories. As a crack extends through a material, the stress concentration along its front gives rise to a "process zone" within which irreversible deformation such as microcracking in concrete or plasticity in metals occurs. Many names, including "structural size", "characteristic length" and "brittleness number" have been given to the parameter $\beta \equiv L/\rho_1$, where L is a characteristic dimension of the structural shape, and ρ_1 is the material-dependent length scale which is proportional to the extent of the process zone that would develop near the front of a very long crack [4–6]. Theoretical and experimental results have shown that $\rho_1 \propto \frac{G_c E}{\sigma_y^2}$, where G_c, E and σ_y represent, respectively, the fracture energy, modulus of elasticity and yield (or ultimate tensile) strength. If β is relatively small, the nominal strength (defined as the maximum load capacity divided by the effective, or uncracked cross-sectional area) of a structure containing a flaw is nearly equal to σ_y, and therefore the structure is referred to as flaw tolerant. In other words, the structure is not penalized for the presence of the flaw beyond a reduction in capacity due to a reduced cross-sectional area. If β is large, however, the structure senses the presence of the flaw, and its nominal strength is much lower than σ_y. The transition from flaw tolerance to flaw sensitivity,

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Nomenclature

δ_{cr}	critical crack opening displacement
δ_{tip}	crack tip opening displacement
ρ_1	characteristic length
σ^{∞}	applied stress
σ_y	yield stress
L	characteristic structural dimension
G_c	fracture energy
E	modulus of elasticity
K_I	stress intensity factor
a	crack length
R	hole radius
D	beam depth

which is driven by the intimate coupling between the structural dimensions and material properties and quantified by β, is a manifestation of the size effect, a name given to the dependence on absolute size of the nominal strength of geometrically similar structures [5]. We note that Bazant's size effect law for quasibrittle materials [5] involves an alternative parameter that accounts for the initial geometry of the cracked structure and thus allows the collapse onto a single curve of experimental data and/or theoretical predictions obtained from different structural geometries. The demonstrations problem presented in this paper does not involve an initial crack, and therefore all results are presented as functions of β.

The simplest quantitative illustration of the ductile to brittle transition is offered by the Dugdale model [6] of the response of a very large two dimensional plate containing a crack of instantaneous length, $2a$, when it is subjected to a uniform tension, σ^{∞} (Fig. 1).

The stress concentration at both crack tips produces a localized region of plastic deformation with extent, ρ, which in approximated by a continuous distribution of springs that transfer a stress σ_y across the top and bottom surfaces of the crack and in turn resist the relative displacement between the crack surfaces. The virtual springs are operative up to a critical stretch, δ_{cr}, at which they rupture. The extent of the plastic (process) zone is such that it eliminates the infinite stresses at the crack tips associated with the elastic solution. This condition requires vanishing of the stress intensity factor, K_I, which for this configuration is written as:

$$K_I = \sigma^{\infty}\sqrt{\pi(a+\rho)} - 2\sigma_y\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right) = 0 \quad (1)$$

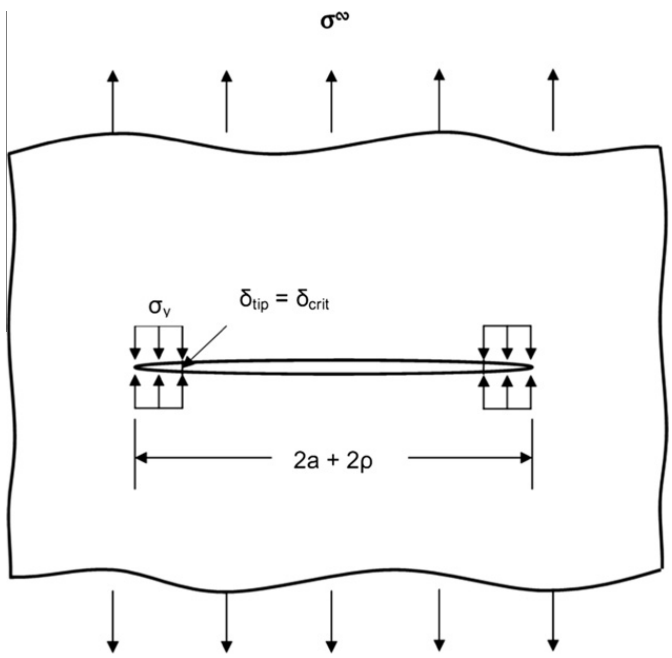

Fig. 1. Dudgale model.

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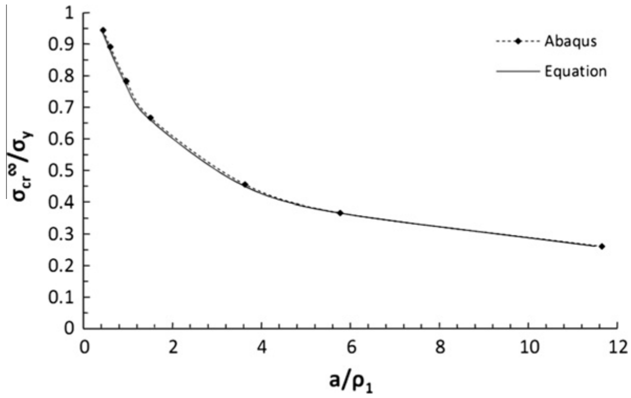

Fig. 2. Normalized strength versus structural size/brittleness predicted by Dugdale model.

where the first term represents the contribution from the applied loading, and the second is due to the crack surface-pinching springs. Eq. (1) provides.

$$\frac{\rho}{a} = \sec\left(\frac{\pi\sigma^{\infty}}{2\sigma_y}\right) - 1 \tag{2}$$

while the normalized opening of the crack at the trailing end of the plastic band, referred to as the crack tip opening displacement (CTOD), is given by:

$$\frac{\delta_{tip}}{a} = \frac{8\sigma_y}{\pi E}\log\left[\sec\left(\frac{\pi\sigma^{\infty}}{2\sigma_y}\right)\right] \tag{3}$$

The normalized strength of the plate, obtained by setting the CTOD equal to δ_{cr}, is obtained as:

$$\frac{\sigma_{cr}^{\infty}}{\sigma_y} = \frac{2}{\pi}\cos^{-1}(e^{-\rho_1/a}) \tag{4}$$

where for this case $\rho_1 = \frac{\pi}{8}\frac{G_c E}{\sigma_y^2} = \frac{\pi}{8}\frac{E\delta_c}{\sigma_y}$ is the extent of the plastic zone in the limit as the crack length approaches infinity. Eq. (4) is plotted in Fig. 2 together with the numerical results obtained using the general purpose finite element code ABAQUS [7]. For this structure the crack length is the only available characteristic dimension, i.e. $L = a$, so that the transition is controlled by the brittleness number a/ρ_1. For small values of brittleness number the normalized length of the plastic zone is relatively large, and the structure is flaw tolerant because the capacity approaches that of a plate with no crack. Large values of brittleness number are associated with relatively small plastic zones, the strength is inversely proportional to the square root of size as predicted by linear elastic fracture mechanics, and the structure is referred to as flaw sensitive.

Next we apply the Dudgale approach to a simple micromechanics model that illustrates how a distribution of interacting flaws can eliminate the flaw sensitivity of structures with relatively large values of β. The motivation of this study comes from two recent and disparate observations. The first involves the failure of the gusset plates that initiated the collapse of the I-35W Bridge in Minneapolis, and the second is the experimentally measured reduction in the size effect exhibited by concrete when it is exposed to very high temperatures.

Fig. 3a is a schematic of the U10 node in the steel truss that contained the infamous gusset plates whose failure initiated the collapse of the Bridge [8,9]. At least two studies [8,9] have suggested that the gusset plates failed because of a design error that caused their cross-sectional area to not be sufficiently large to resist the forces it experienced on the day of the collapse; if the thickness of the gusset plates had been twice as large the Bridge would still be standing. The blue lines in the drawing represent the cracks that were observed in one of the gusset plates recovered from the wreckage. Fractographic analyses determined that the coalescence of the rivet holes contained within the gusset plate and shown in Fig. 3b [9] resulted from tension and shear induced ductile tearing of the ligaments between the holes. No evidence was found of brittle cracking or subcritical fatigue crack growth. An interesting question is whether plates similar to these containing closely spaced R-diameter rivet holes and subjected to monotonically increasing forces, would be associated with a size effect if they are made of a steel with very small values of ρ (if their toughness (yield strength) was lower (higher)), i.e. if β is significantly larger. The micromechanics model described subsequently suggests that perforated holes are rendered flaw tolerant when the spacing between the holes decreases. It is important to note that in no way do we suggest that cracking could not develop is plates containing closely spaced holes. High cycle fatigue is certainly a possibility. But here we focus on static strength.

Fig. 4a shows the diffuse cracking and porosity that develops within the microstructure of concrete after it is heated to 600 °C [10]; such damage was not observed in similar specimens exposed to 20 °C and 300 °C. Fig. 4b presents the nominal strength of geometrically similar notched beam specimens of varying depth, D, that were loaded in three-point bending to failure after the heat treatments. It was determined by fitting the experimental data to fracture mechanics simulations and

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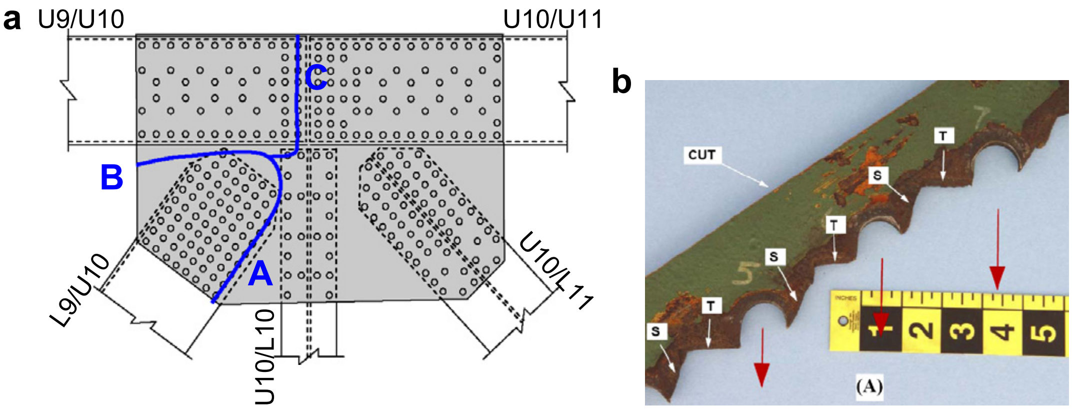

Fig. 3. (a) Node U10 of I-35W Bridge showing gusset plate connecting truss members. The blue lines labeled A, B, C represent the observed cracking patterns within the gusset plates. A and C indicate cracks that linked the rivet holes. (b) Part of the gusset plate indicating the ductile tearing of the ligaments between the rivet holes; *S* and *T* represent shear and tension, respectively (Refs. [8,9]).

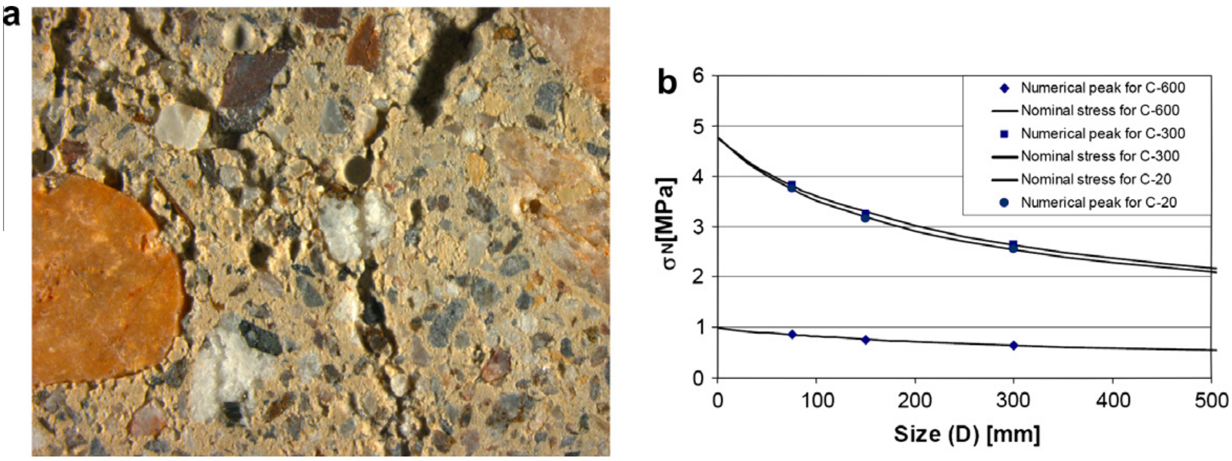

Fig. 4. (a) Micrograph of concrete microstructure after exposure to 600 °C. (b) Normalized strength of concrete at 20 °C, 300 °C and 600 °C as functions of specimen dimensions. (Ref. [10]).

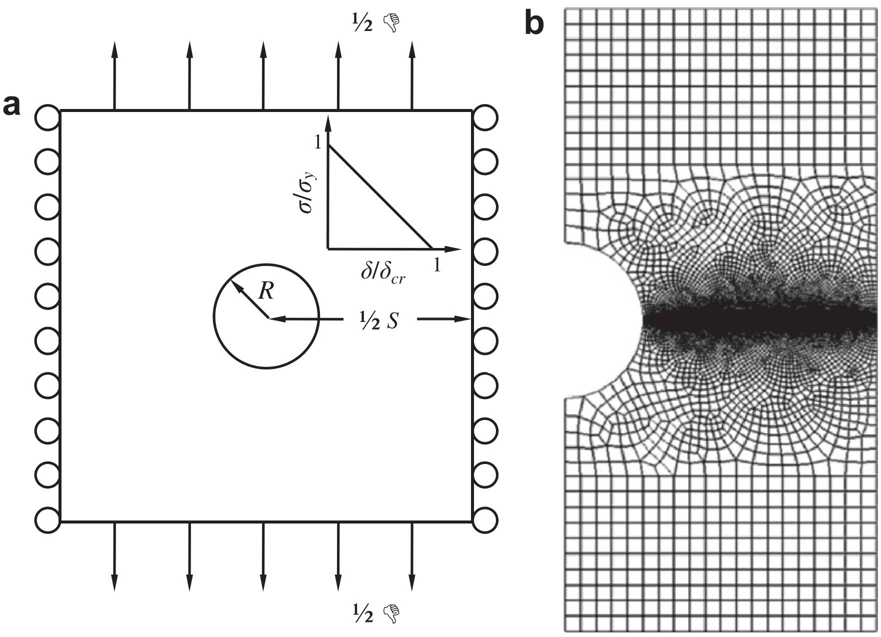

Fig. 5. (a) Micromechanics model. (b) Finite element discretization of micromechanics model.

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Bazant's size effect law that the lowest two temperatures are consistent with ρ_1 = 138 mm (20 °C) and ρ_1 = 152 mm (300 °C), and qualitatively similar to the curve shown in Fig. 2. The highest temperature, however, exhibited a very weak size effect (a flattening of the curve within the typical range of brittleness number) consistent with a much larger ρ_1 = 700 mm. What is remarkable about the strength of the concrete exposed to the highest temperature is how much less its strength in the presence of the notch is penalized when its structural dimensions increase. The flattening of the strength–size curve is a result of the complex interaction between the initial notch created in the specimen and the surrounding material. There are two possible contributions to the reduction in notch sensitivity at the highest temperature. The first involves the toughening produced by the interaction of the closely spaced pores that are absent at the lower temperatures, and the second is the chemical transformation of calcium silica hydrates that renders the matrix soft. However, both effects can be understood qualitatively by a reduction in stiffness within a volume, using the simplified micromechanics model presented next.

Consider a periodic arrangement of holes spaced at distance, S, within an infinitely extended plate (Fig. 5a). It is assumed that ultimate failure results from the coalescence of Dugdale-type cracks that initiate and extend from the edges of the hole. The commercial finite element method code ABAQUS was used to calculate the maximum stress achieved under a monotonically increasing applied displacement, Δ. A representative finite element discretization of the plate is shown in Fig. 5b. Without loss of generality a linear softening stress-crack opening relation typical of concrete, $\sigma/\sigma_y = (1 - \delta/\delta_{cr})$, was used to determine the maximum nominal stress achieved as functions of spacing of the holes and R/ρ_1 (inset of Fig. 5a).

For this example, $\rho_1 = \frac{G_c E}{\sigma_y^2} = \frac{1}{2}\frac{E\delta_c}{\sigma_y}$. Representative load-carrying capacities and nominal stress–nominal strain curves are shown in Figs. 6 and 7. Fig. 6 shows that the size effect is drastically reduced with decreasing hole spacing, regardless of the intrinsic brittleness of the material that comprises the perforated plate. For the closest spacing computed, R/S = 0.333, the normalized strength is reduced by 10% at most across the wide range of R/ρ_1. The stress profiles, which are not included here for the sake of space limitation, showed that for the isolated hole the process zone that fully develops near peak load is relatively small compared to the uncracked ligament, and that it propagates in a steady-state across and unzips the ligament. For the smallest spacing, however, the length of the process zone upon its full development near peak load is nearly equal to the length of the uncracked ligament. In essence the closest spacing configuration achieves net section yield (albeit with a strain softening stress-separation law).

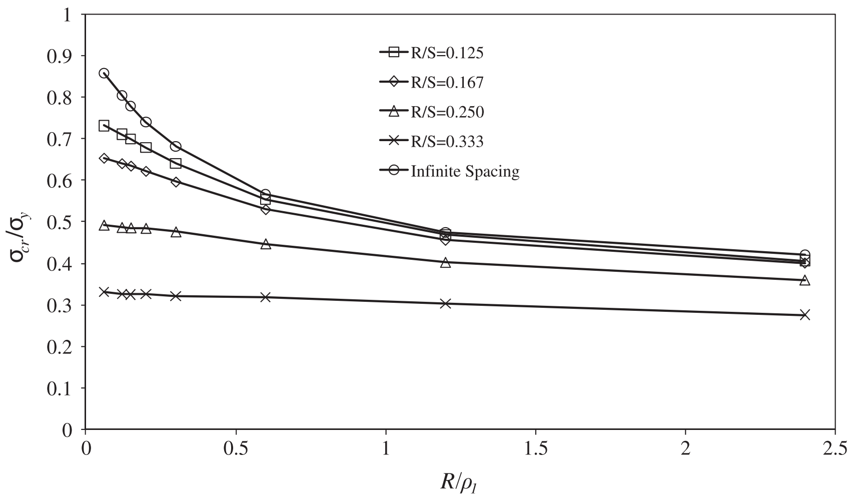

Fig. 6. Nominal strength of plate with periodic arrangement of holes as functions of brittleness.

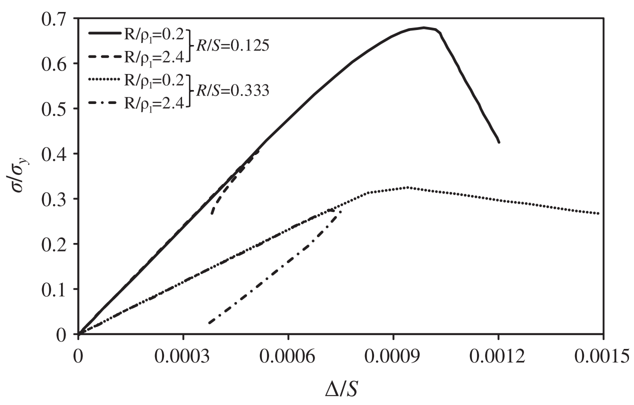

Fig. 7. Nominal stress–nominal strain curves of plate with periodic arrangement of holes for selected values of spacing and brittleness number.

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Fig. 7 shows how the plate's ductility is influenced by hole spacing. As expected, the response of a plate containing an isolated hole ($R/S = 0.125$) is ductile (significant energy dissipation post-peak) for relatively small brittleness number ($R/\rho_1 = 0.2$), but exhibits a steep snap-back brittle response (this curve was truncated because of numerical instability) for large brittleness number ($R/\rho_1 = 2.4$). However, for the case of closely spaced holes ($R/S = 0.333$), the post-peak behavior exhibited by the isolated hole at the larger value of brittleness number is associated with a much larger amount of energy dissipation as reflected in the stress–strain curve. We note that for this specific flawed plate configuration the response remains unstable under displacement control, but the increased ductility with decreased spacing is clearly illustrated.

3. Discussion

The simple pedagogical qualitative model presented here has shown that the interaction between flaws can render structures comprised of very brittle materials flaw tolerant, notch insensitivite, and ductile. This insight, which was not the focus of most previous studies that focused on the response of structures containing isolated flaws, can guide the design of structures comprised of inherently brittle materials.

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